

Introduction to Bioinformatics for Computer Scientists

Lecture 6

Plan for next lectures

- Today: Introduction to phylogenetics
- Lecture 7 (Alexis): Phylogenetic search algorithms

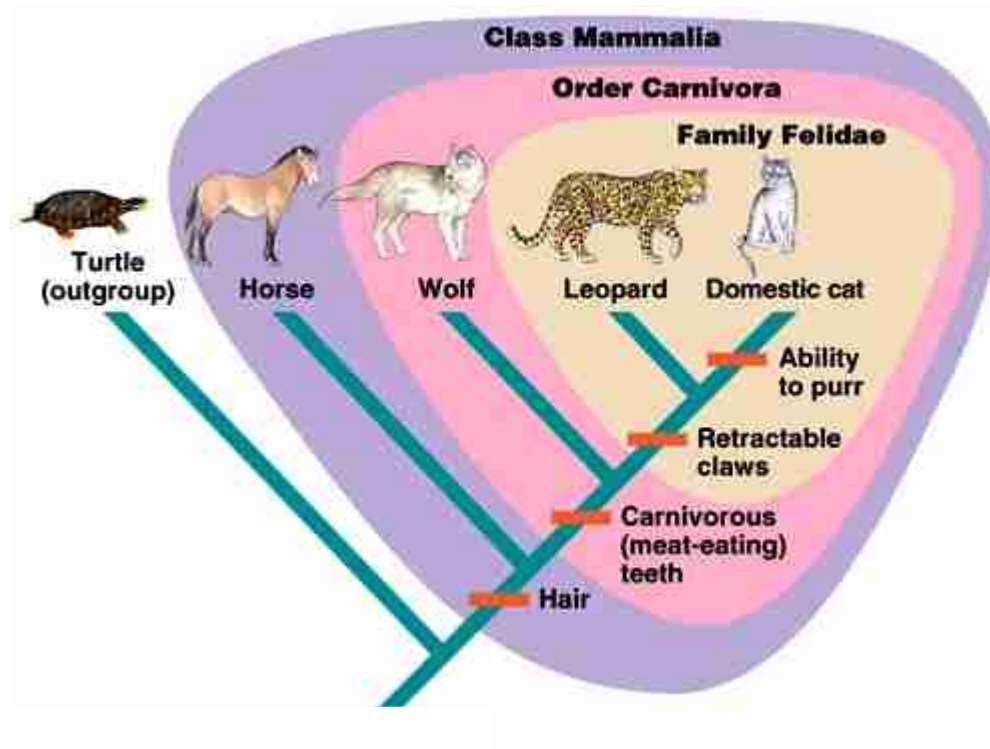
The story so far

- Biological Terminology: RNA, DNA, genes, genomes, etc
- Pair-wise Sequence Alignment
- Sequence Comparison
- Genome Assembly
- Multiple Sequence Alignment

The story so far

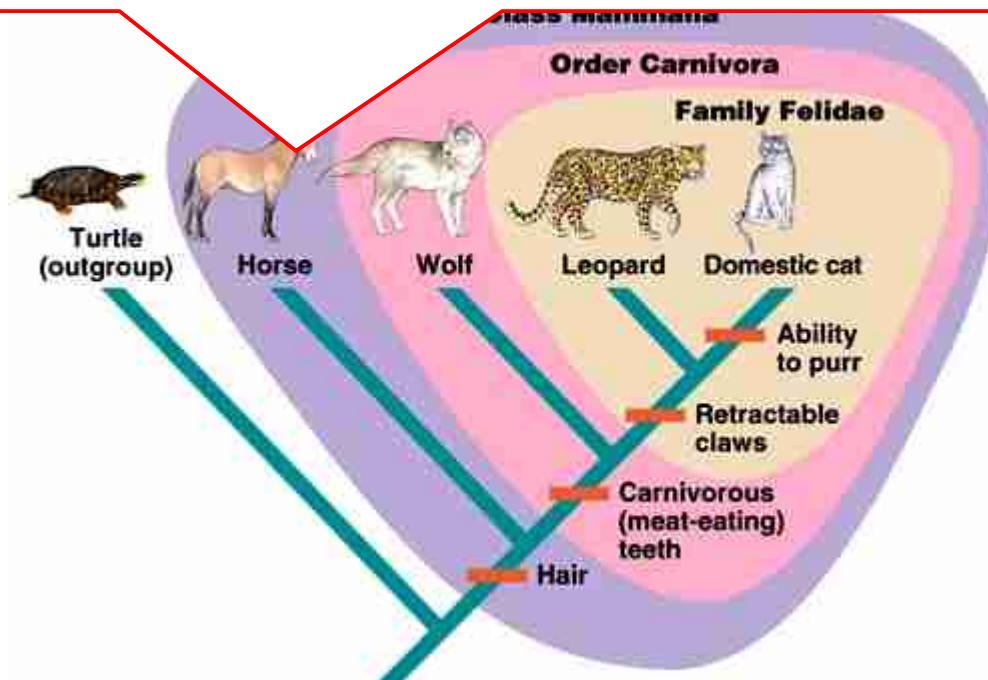
- Biological Terminology: RNA, DNA, genes, genomes, etc
- Pair-wise Sequence Alignment
- Sequence Comparison
- Genome Assembly
- Multiple Sequence Alignment
- **Phylogenetic Inference**

A Taxonomy



A Taxonomy

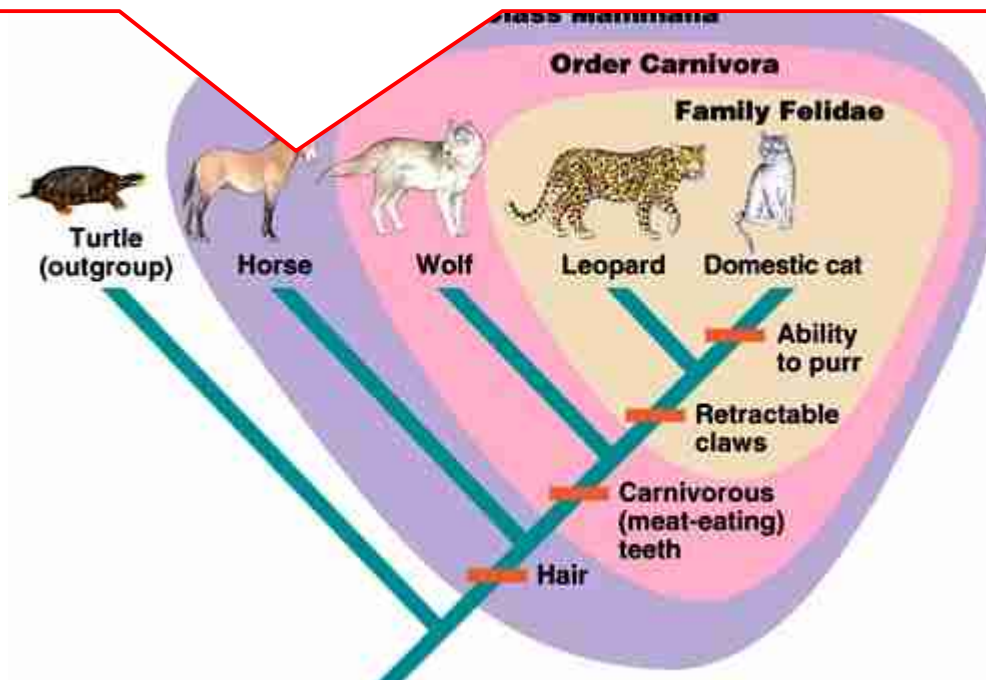
First systematic classification of living beings by Aristotele 384 -382 BC
Some terms still in use today, e.g., classification of animals into *Vertebrates* versus *Invertebrates*



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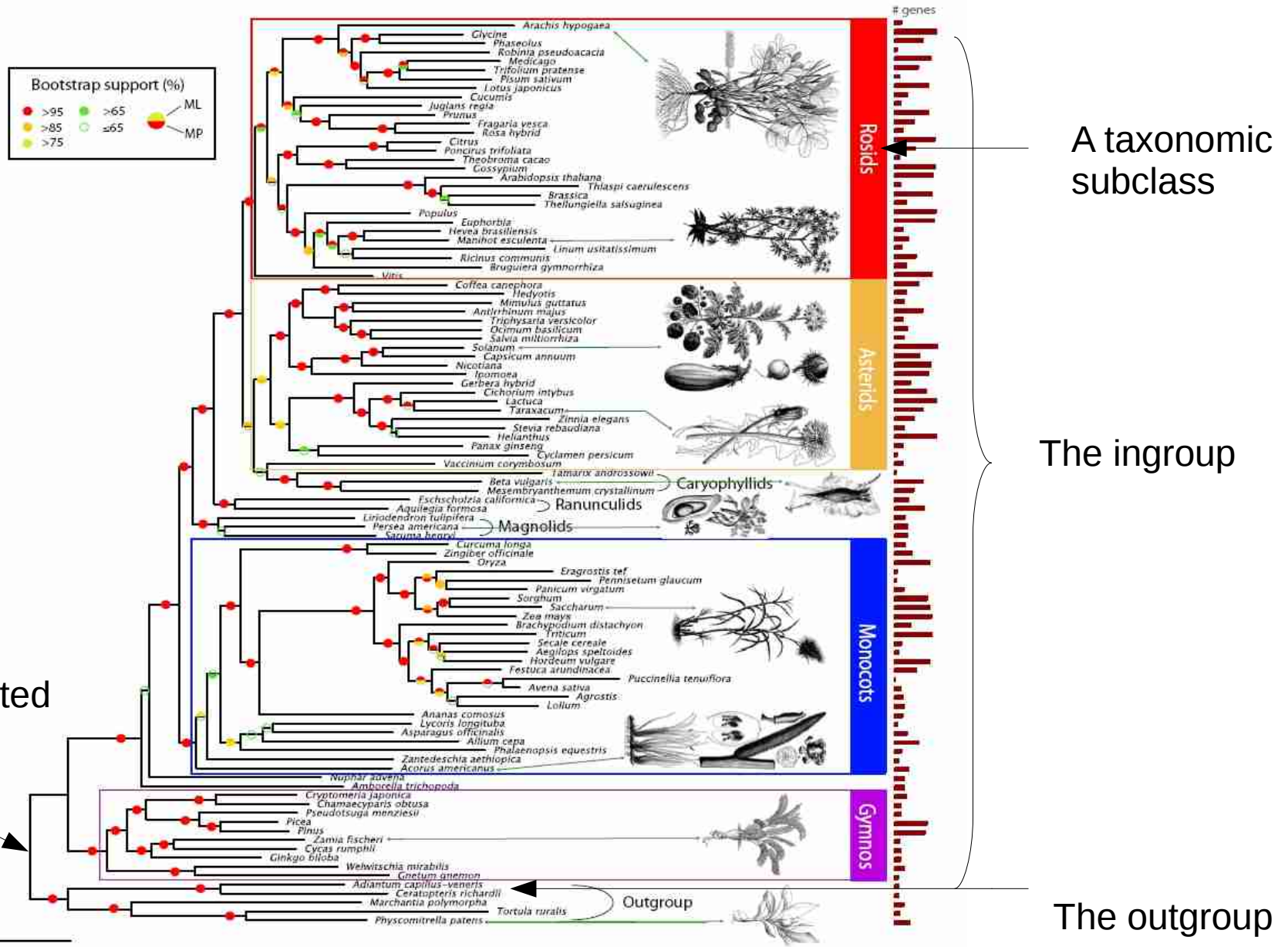
Wirbeltiere



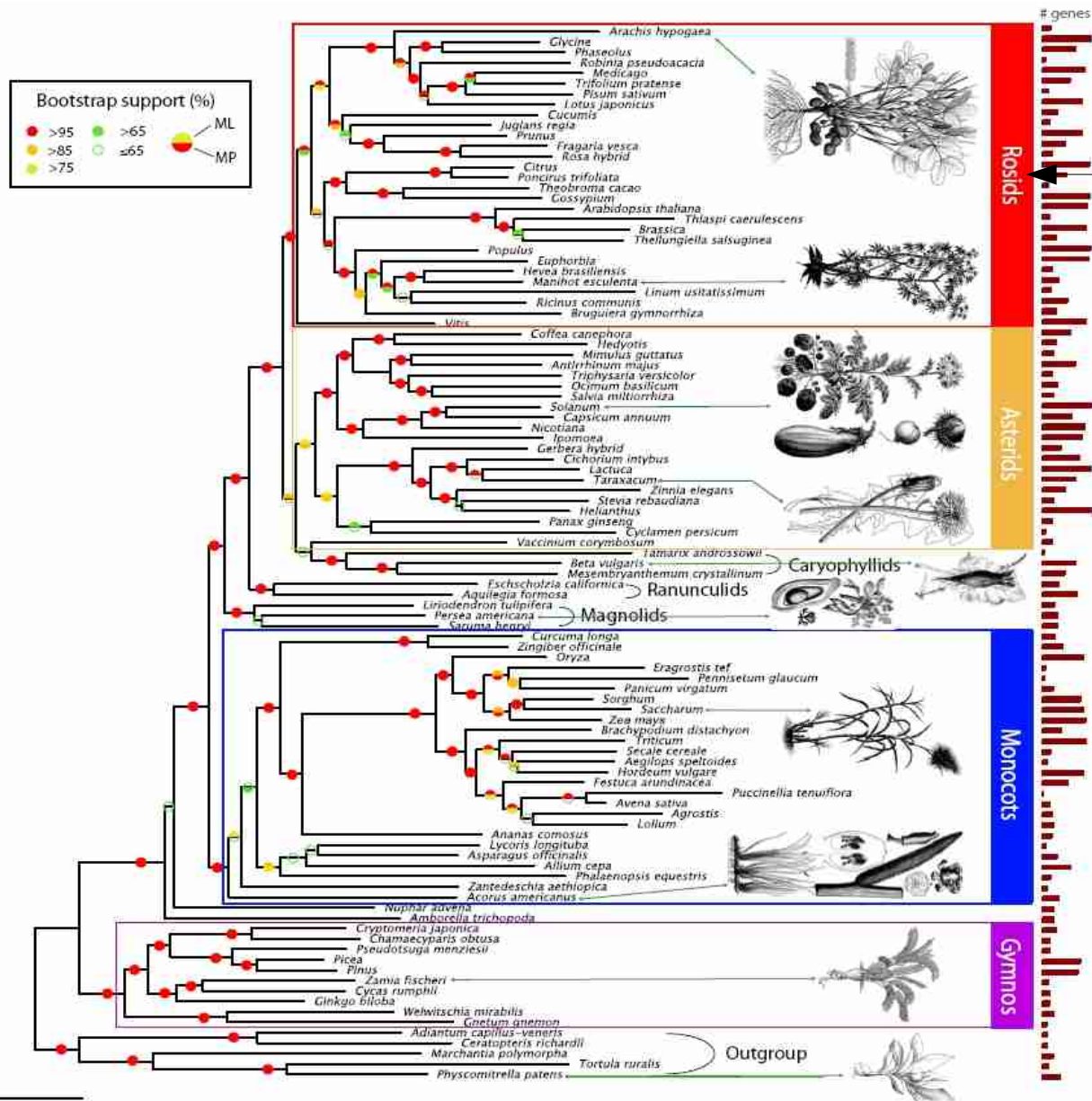
Taxonomy

- Group biological organisms (species) into groups with similar characteristics
- Define characteristics of groups at different hierarchy levels, e.g., animals > mammals > great apes
- Taxonomic ranks
 - Domain → three domains of life
 - Kingdom
 - Phylum
 - Class
 - Order
 - Family
 - Genus
 - Species

A Phylogeny or Phylogenetic Tree



A Phylogeny or Phylogenetic Tree



In Phylogenetics such a subtree is often also called *Lineage!*

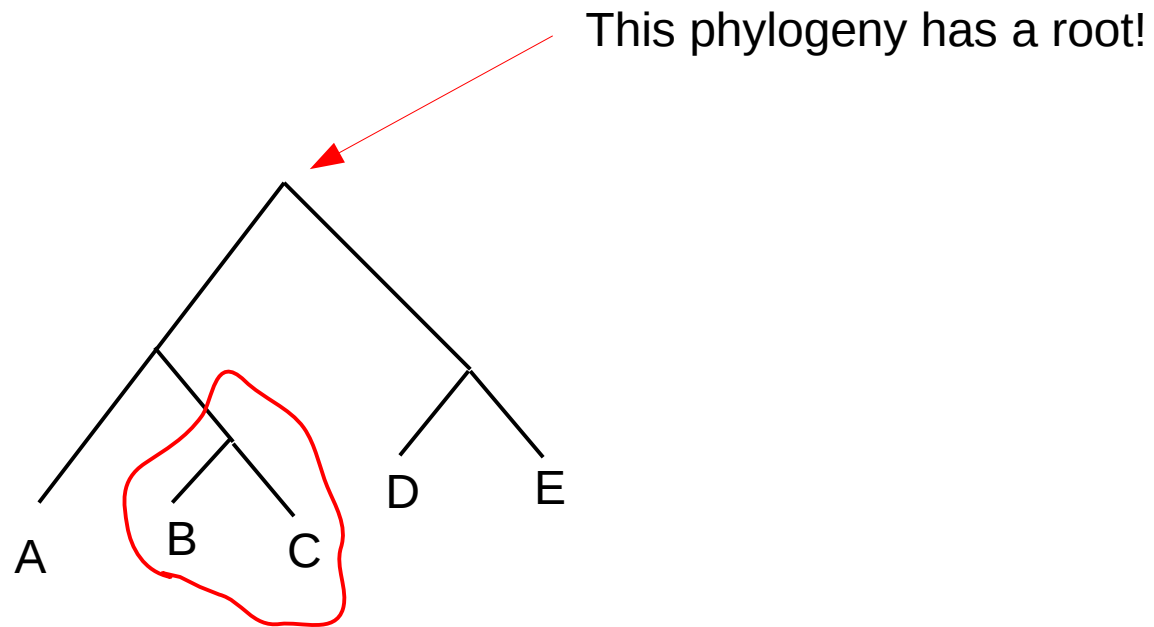
Phylogeny

- An unrooted strictly binary tree
- Leafs are labeled by *extant* “übrig geblieben” (currently living) organisms represented by their DNA/Protein sequences
 - we can also sequence ancient DNA, see, for instance, the neandertal genome: “The complete genome sequence of a Neanderthal from the Altai Mountains”, *Nature* 2013
 - depends on temperature, time, and other environmental conditions
 - up to 300,000 years back, see
<http://www.pnas.org/content/110/39/15758.abstract>
- Inner nodes represent *hypothetical common ancestors*
- *Outgroup*: one or more closely related, but different species → allows to root the tree

Taxon

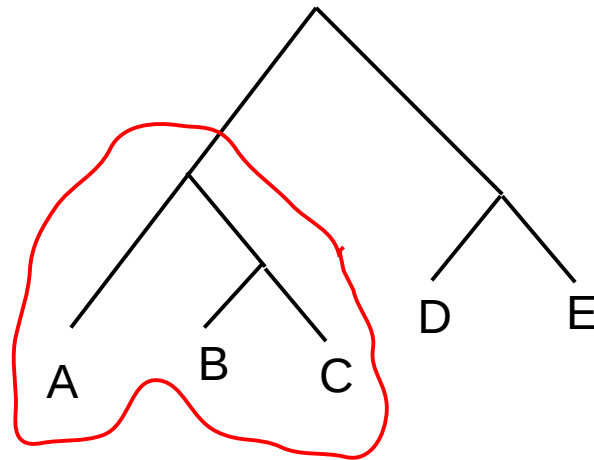
- Used to denote clades/subtrees in phylogenies or taxonomies
- A group of one or more species that form a biological unit
- As defined by taxonomists
 - subject of controversial debates
 - part of the culture/fuzziness of Biology
- In phylogenetics we often refer to a single leaf as taxon
 - the plural of taxon is *taxa*
 - we often say that a tree with n leaves (sequences) has n taxa

Some more terminology



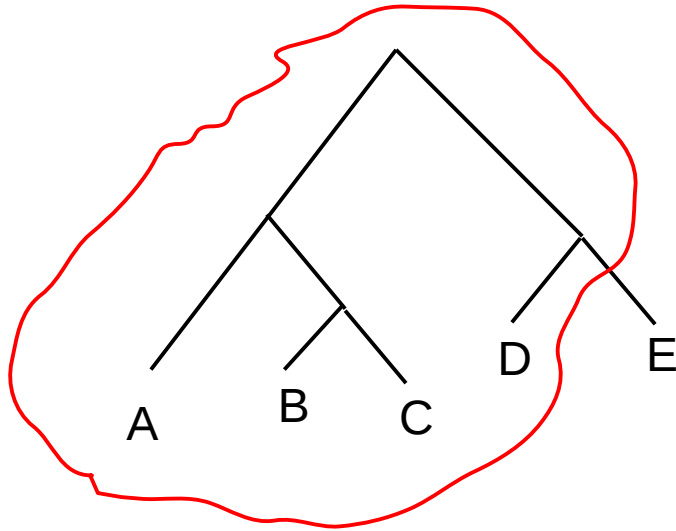
B and **C** are a *monophyletic* group; they are sister species

Some more terminology



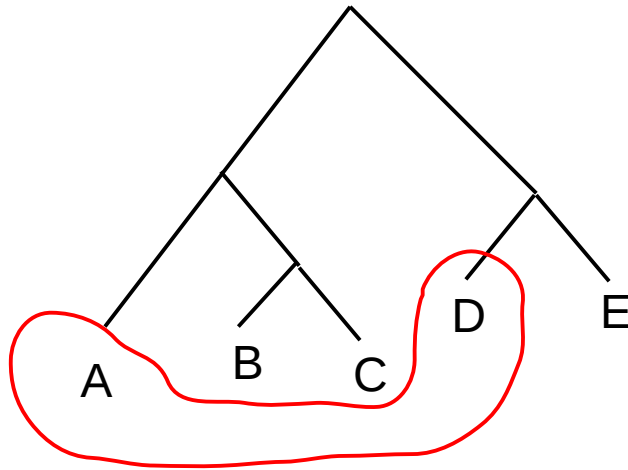
(A,B,C) is a *monophyletic* group; it is sister to **(D, E)**

Some more terminology



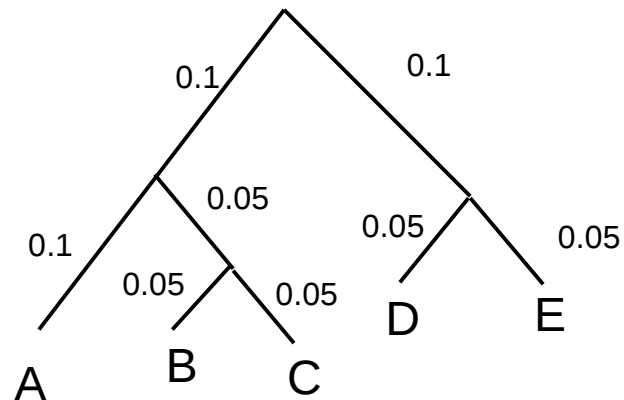
(A,B,C,D) is *paraphyletic* → **E** is excluded

Some more terminology



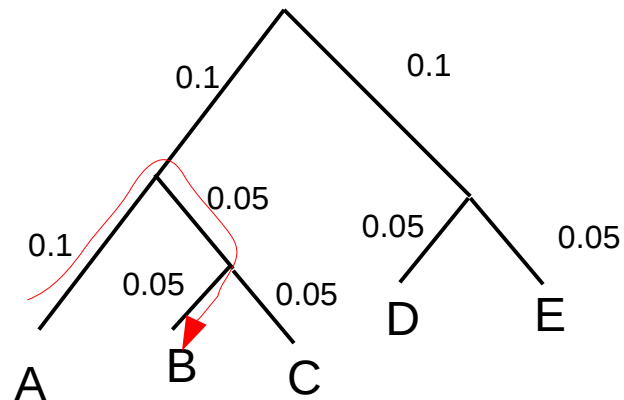
(A,D) is a *polyphyletic* group → their most recent common ancestor (MRCA) is excluded

Some more terminology



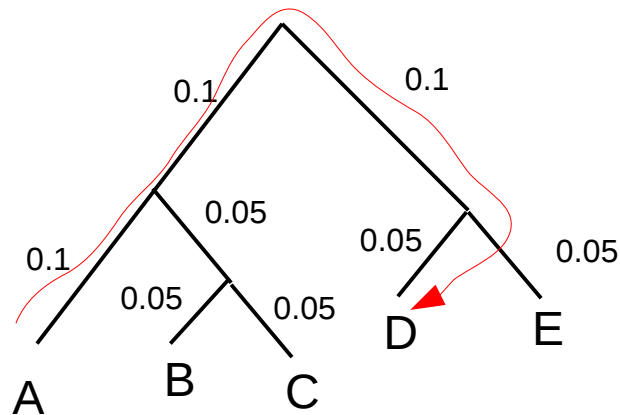
Tree-based or *patristic distance* between two taxa:
Sum over branch lengths along the path in the tree, e.g.:

Some more terminology



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Sum over branch lengths along the path in the tree, e.g.:
A ↔ **B**: 0.2

Some more terminology



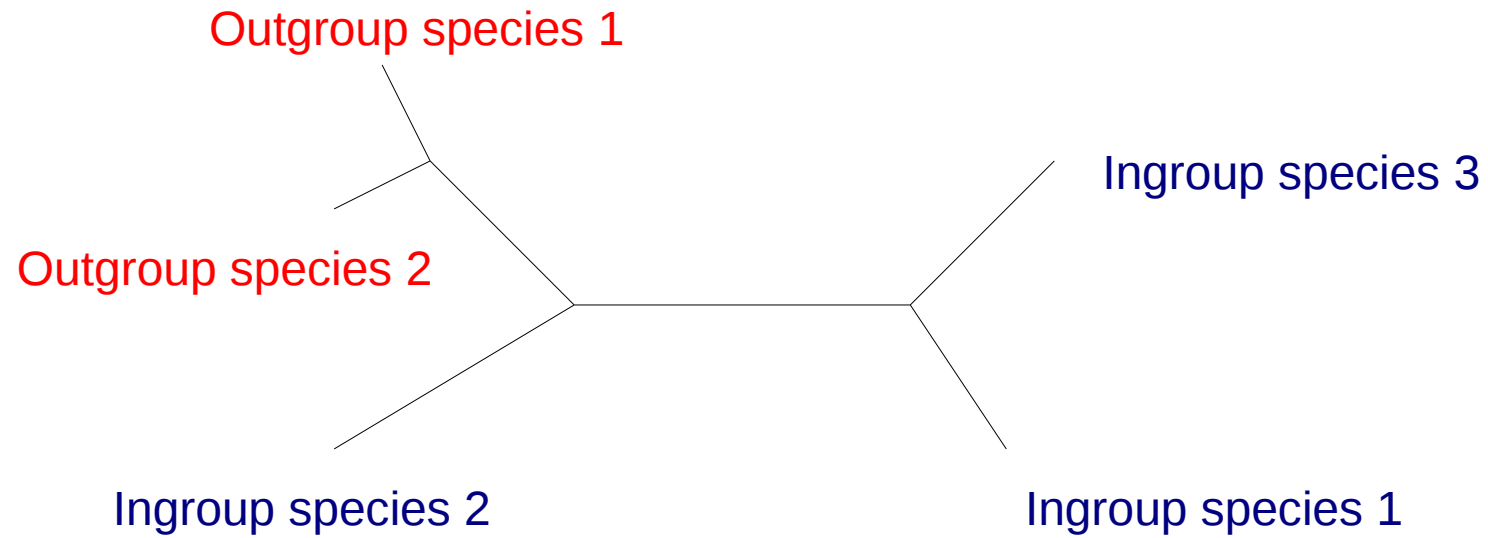
Tree-based or *patristic distance* between two taxa:

Sum over branch lengths along the path in the tree, e.g.:

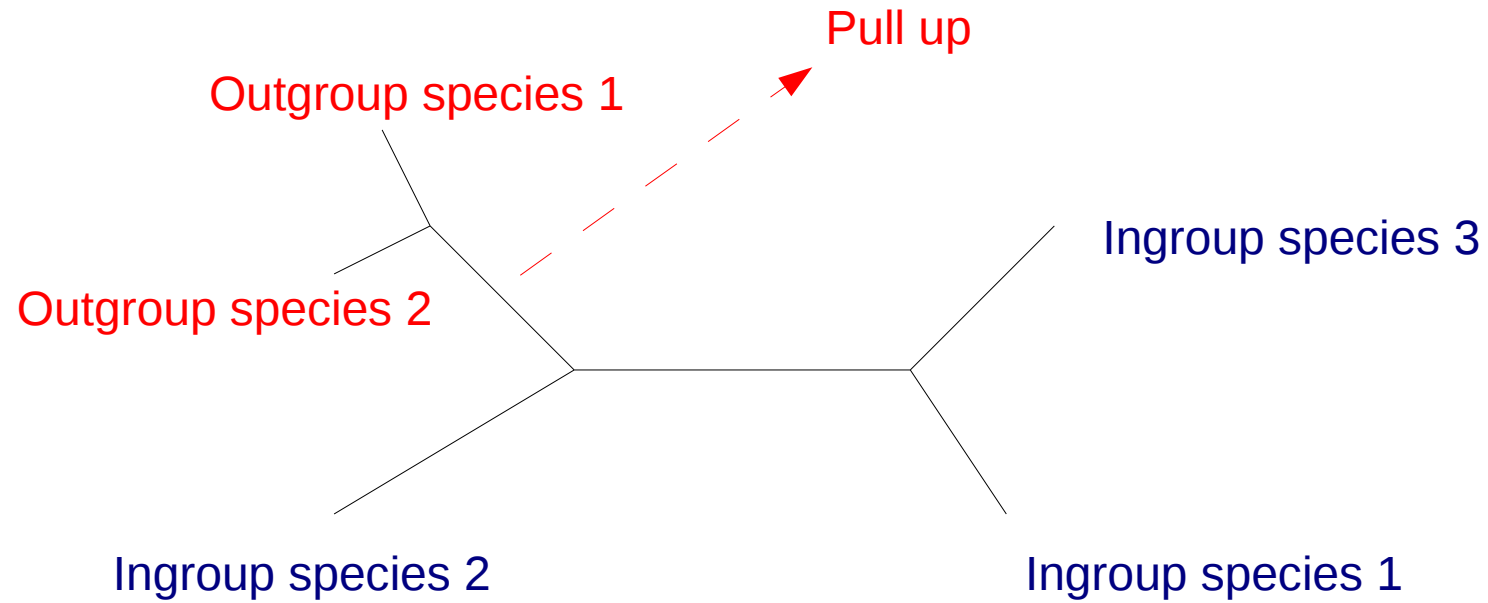
A ↔ **B**: 0.2

A ↔ **D**: 0.35

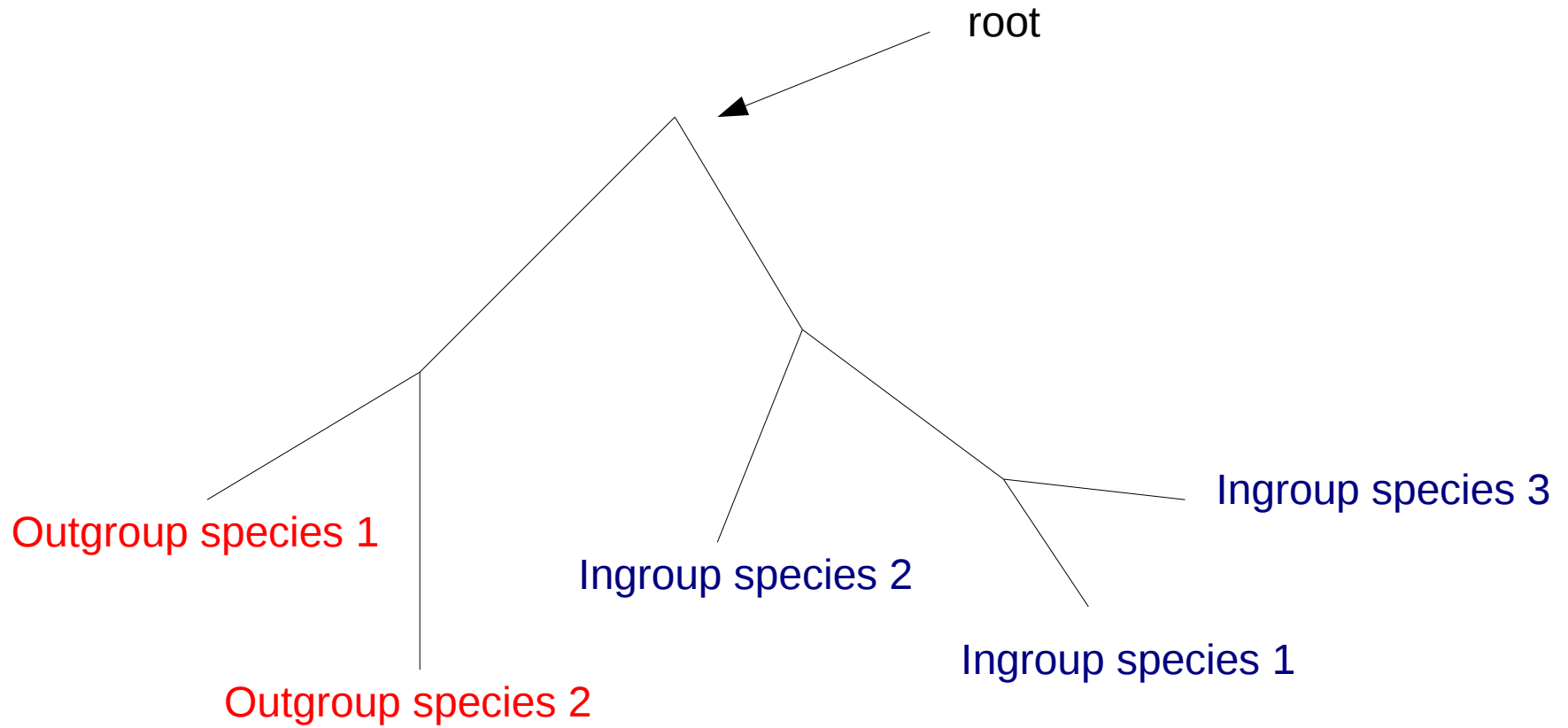
Tree Rooting



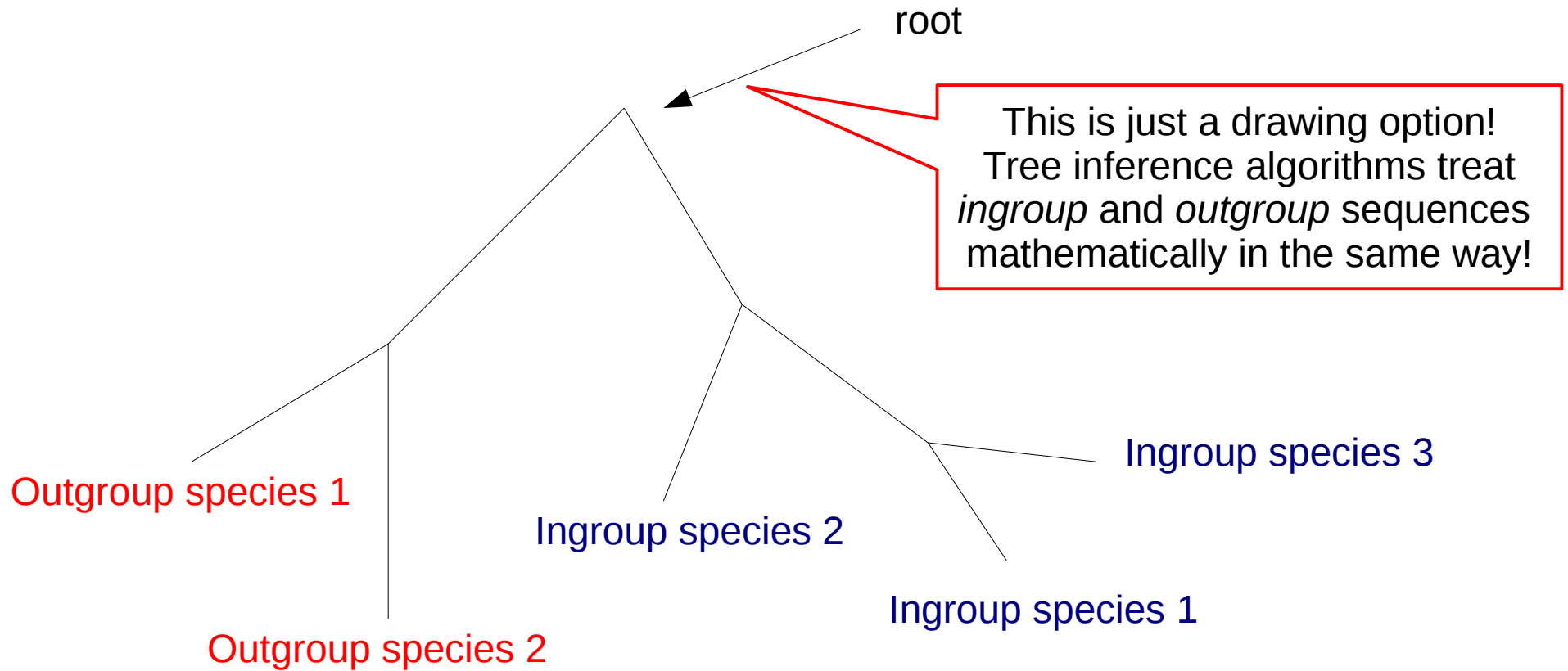
Tree Rooting



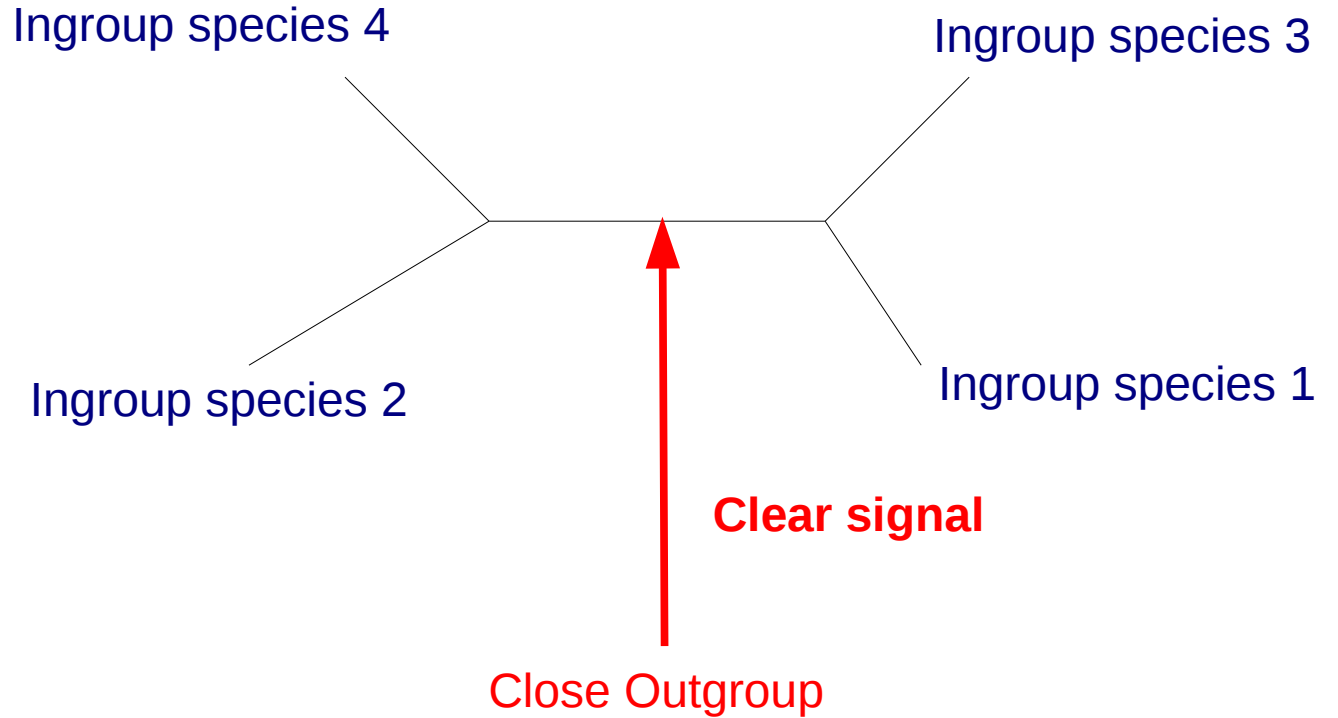
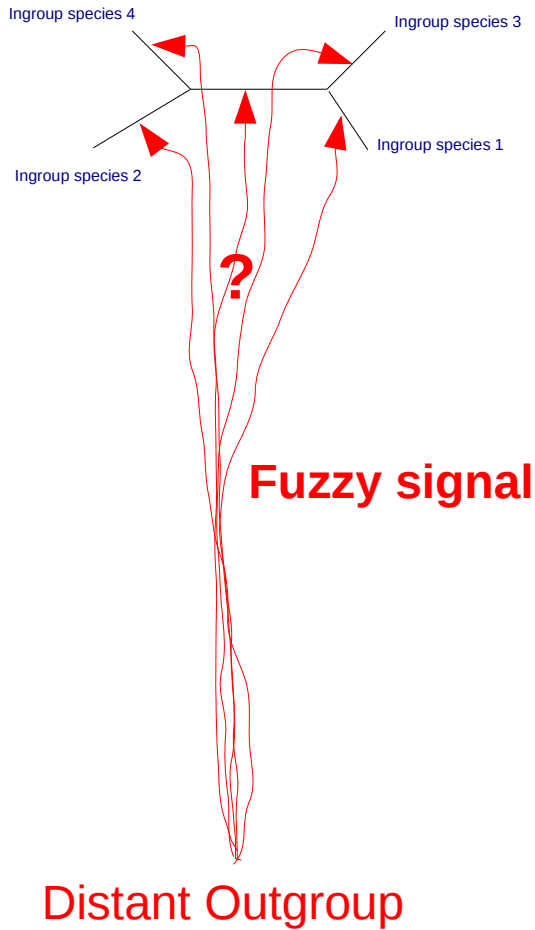
Tree Rooting



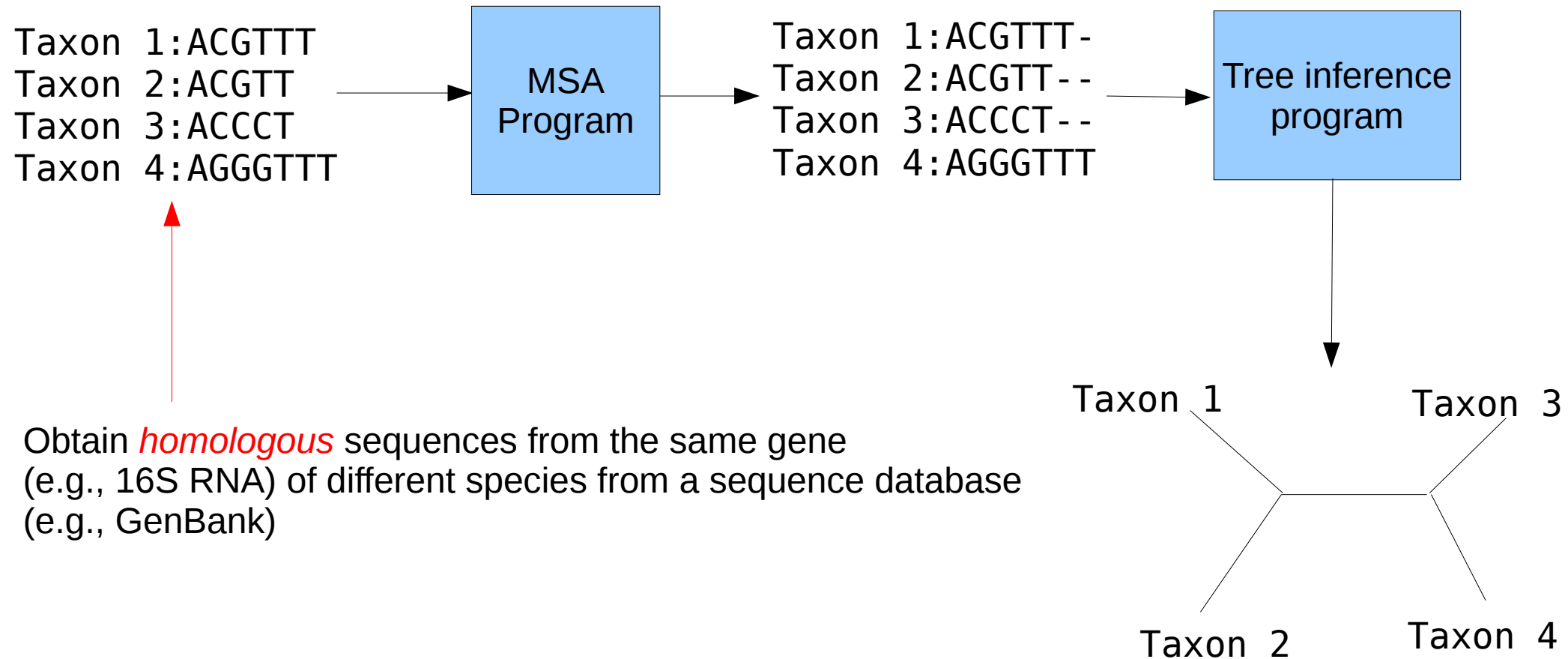
Tree Rooting



Outgroup Choice

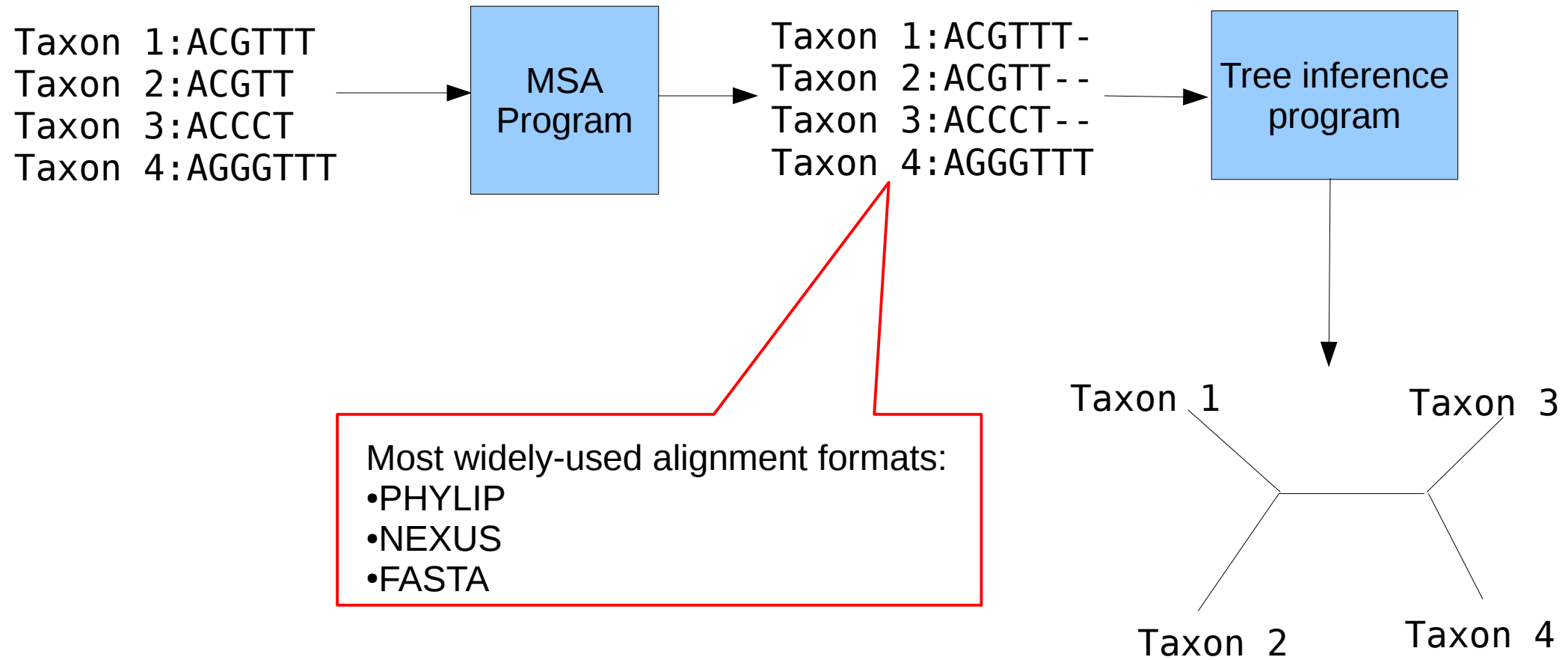


Tree Inference

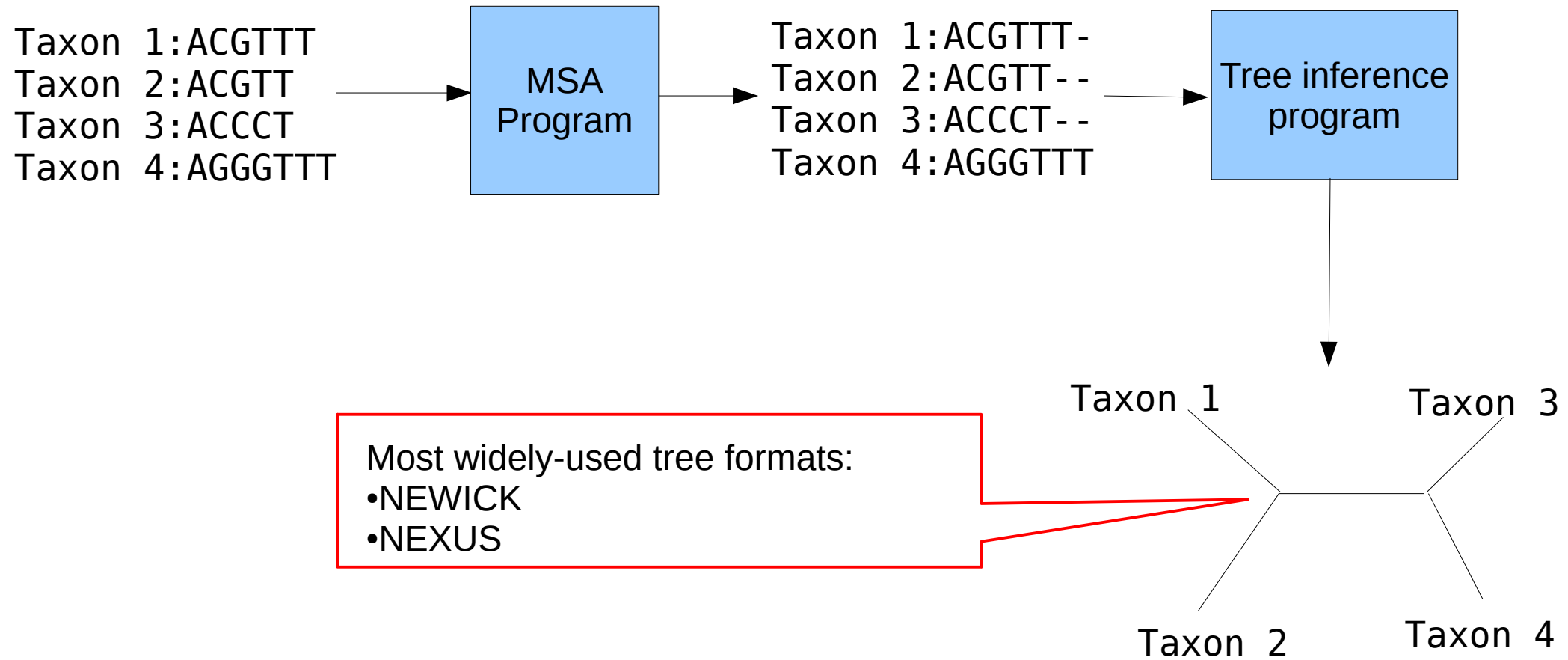


Obtain *homologous* sequences from the same gene (e.g., 16S RNA) of different species from a sequence database (e.g., GenBank)

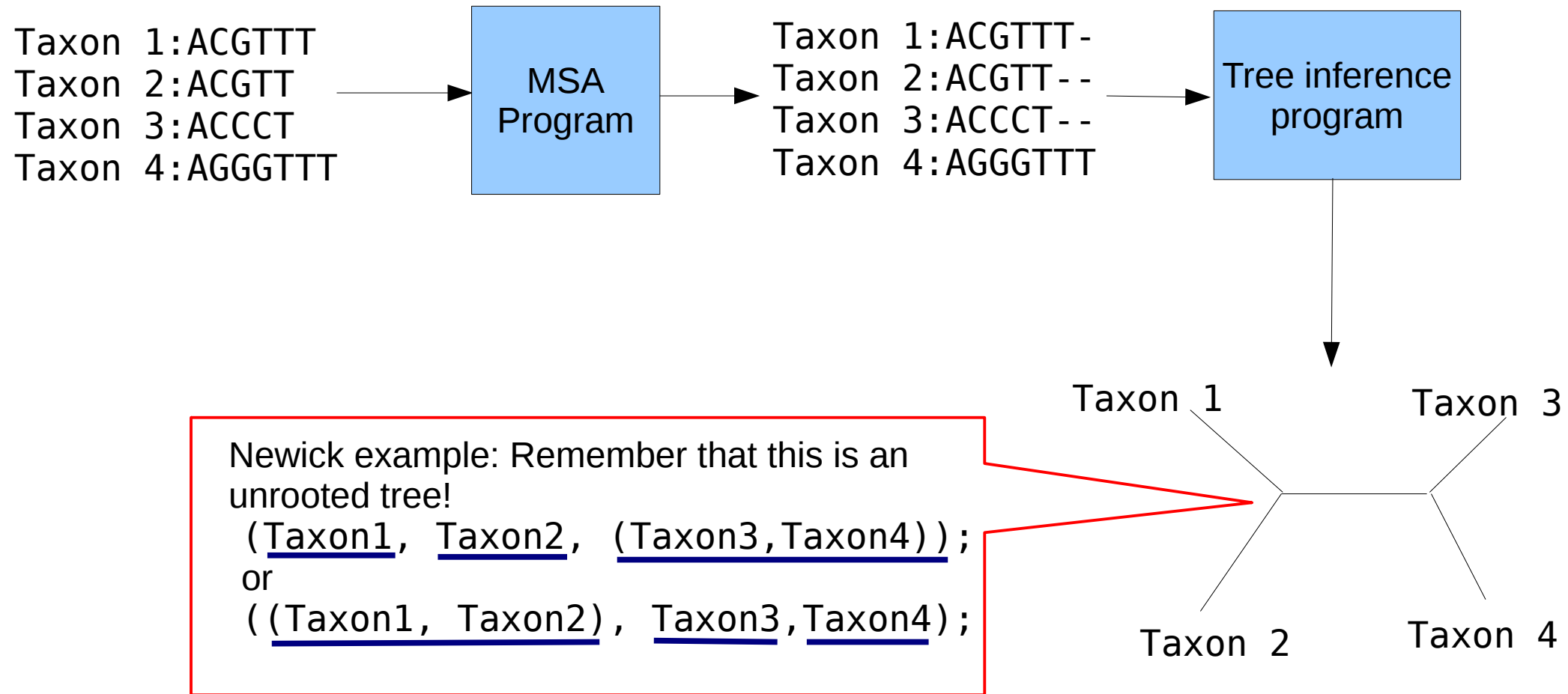
Tree Inference



Tree Inference

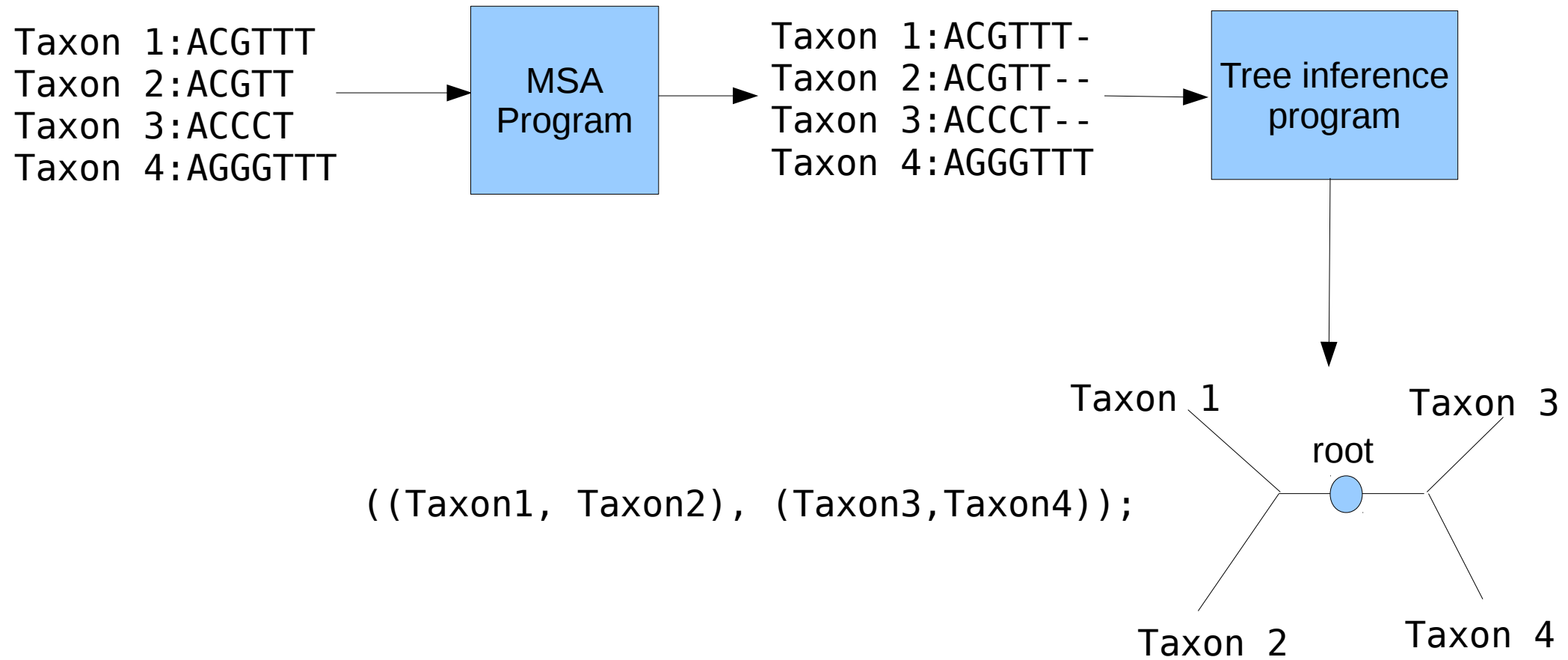


Tree Inference

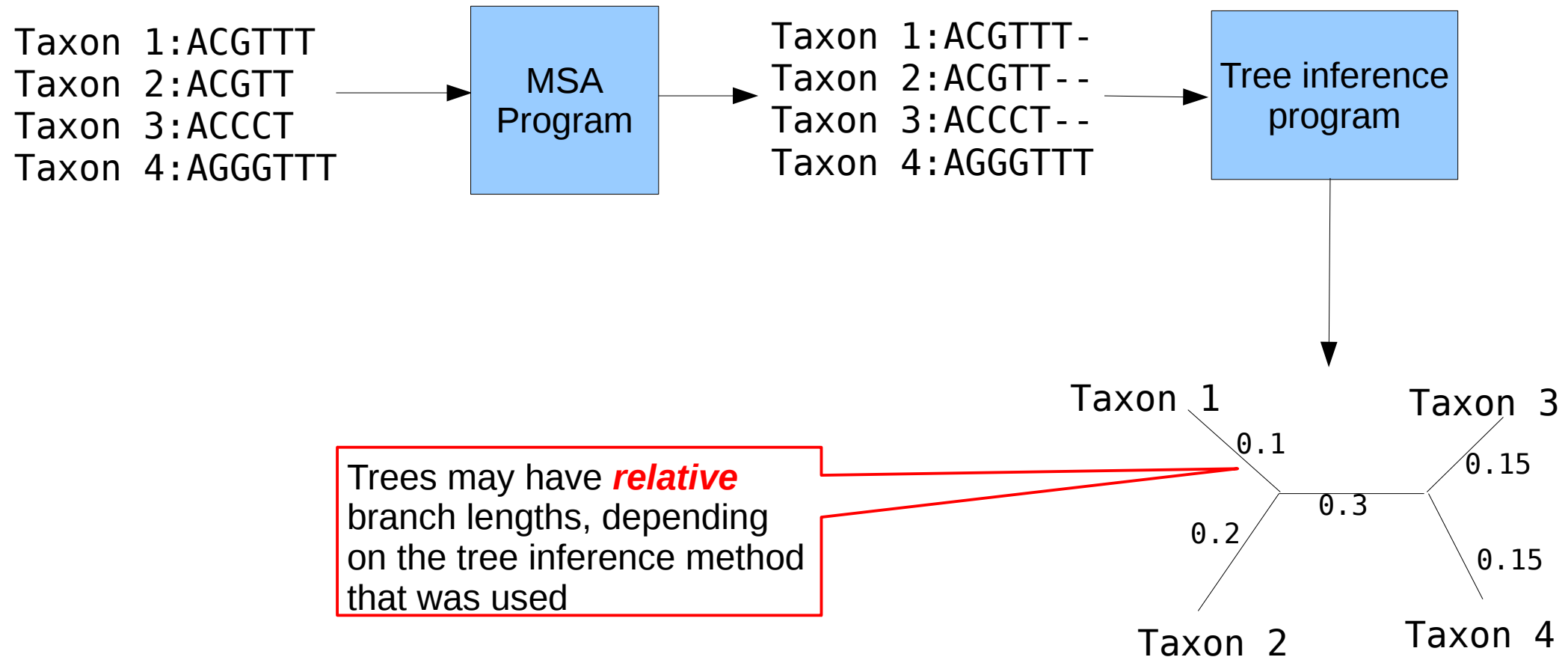


Top level trifurcation

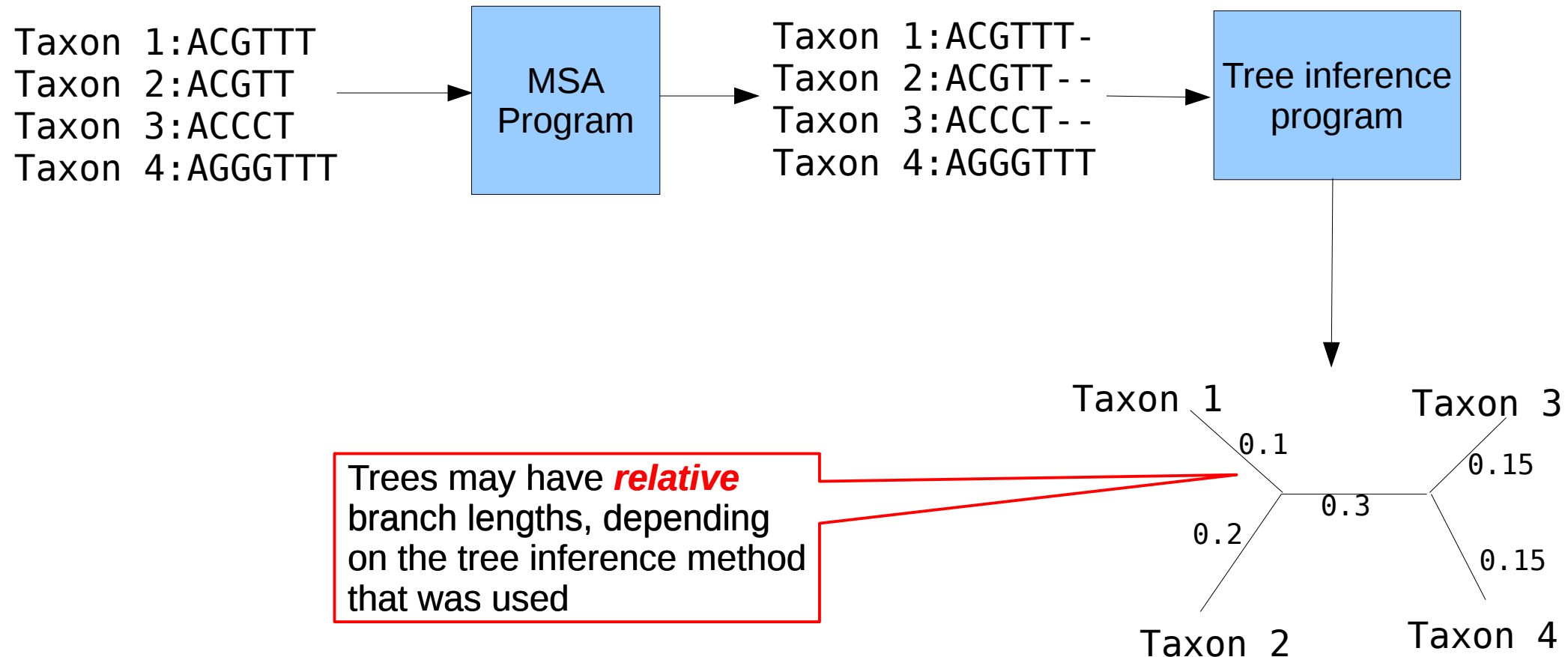
Tree Inference



Tree Inference



Tree Inference



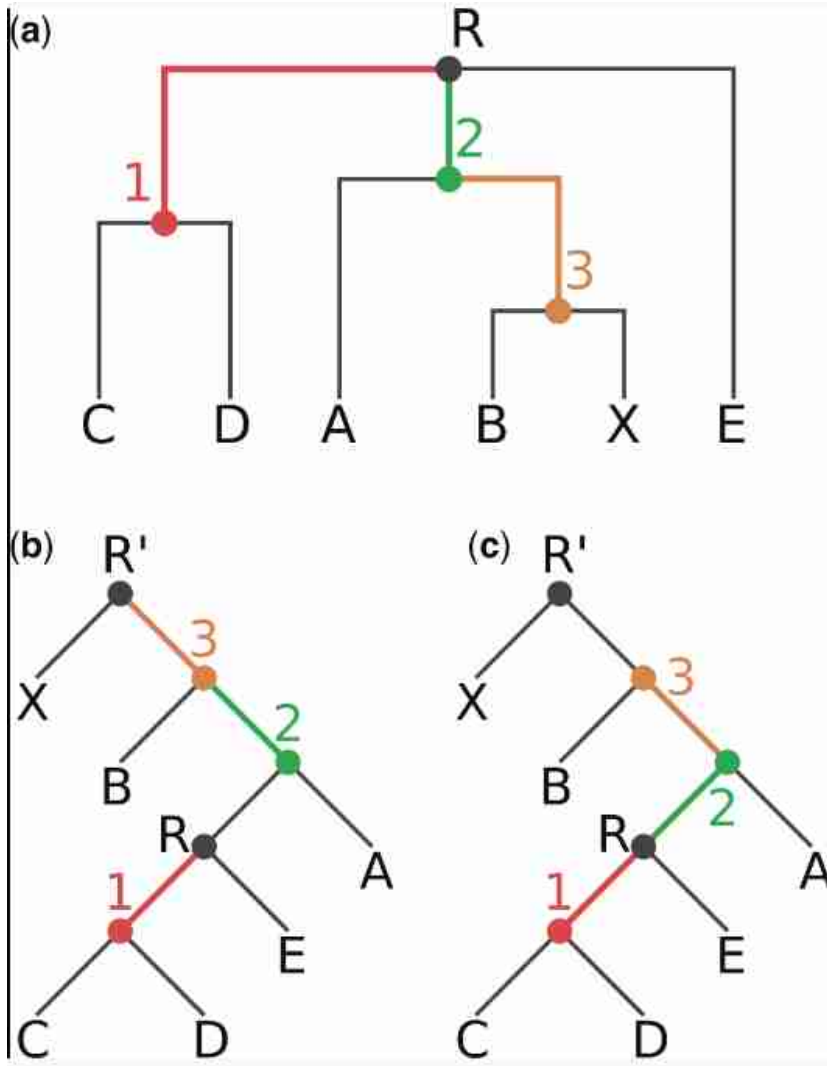
Newick format with branch lengths:

(Taxon1:0.1,Taxon2:0.2,(Taxon3:0.15,Taxon4:0.15):0.3);

Problems with Newick tree format

- Except for branch length values: no way to associate meta-data to branch lengths
- However, there is important meta-data, e.g., branch support: how well is a branch in the tree supported?
 - ad hoc solution: represent branch support values as node meta-data!
 - this causes problems

Problems with Newick tree format



Branch support values represented as node meta-data can be assigned incorrectly to branches after re-rooting.

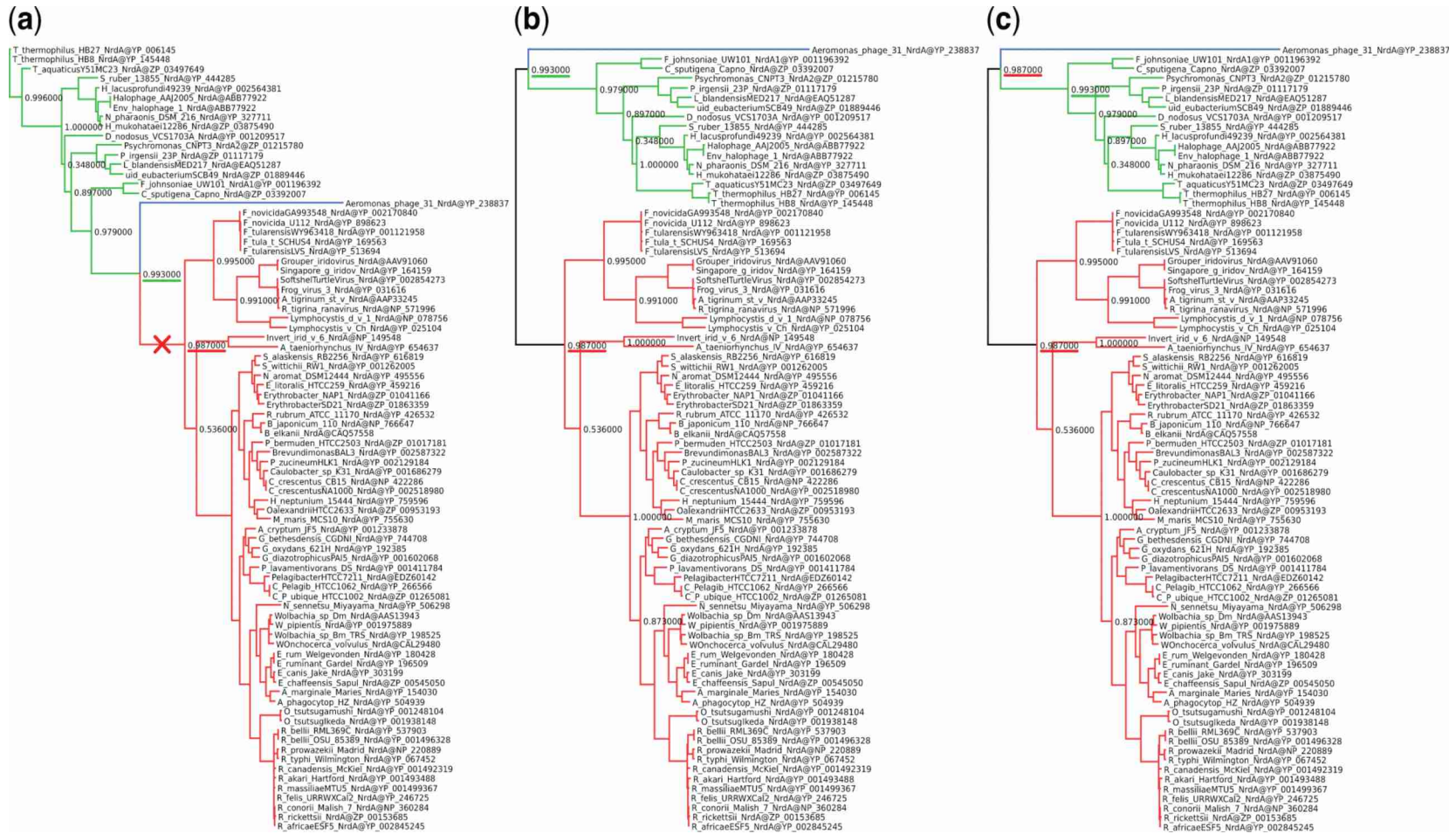
About 50% of the tools we checked had this Problem. For details see:

<https://academic.oup.com/mbe/article/34/6/1535/3077051>



Which representation is correct?

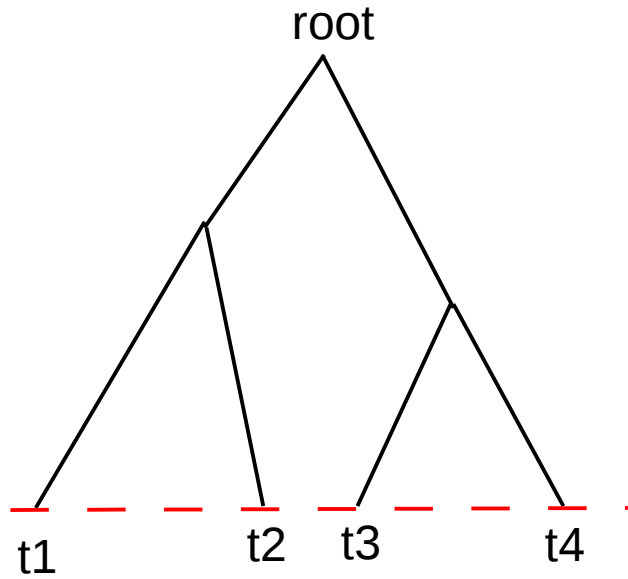
A real example



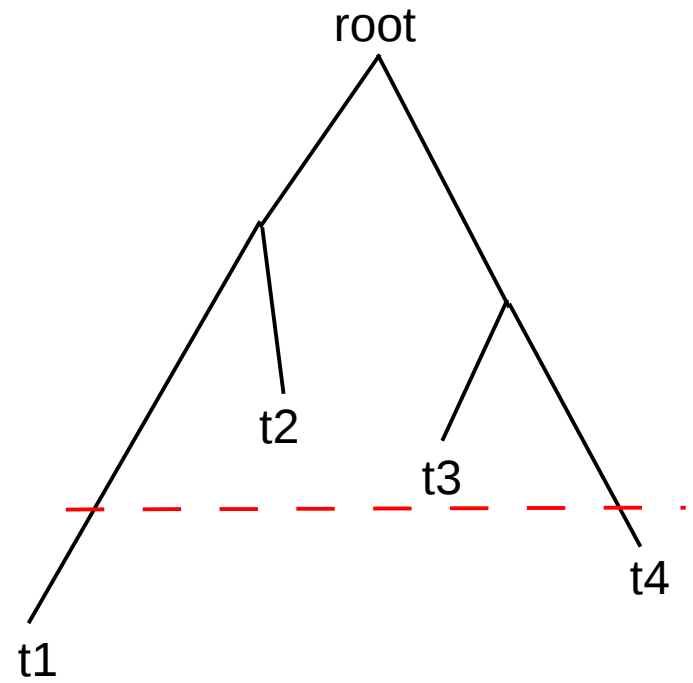
a) original tree
 b) re-rooted tree with shifted support values
 c) re-rooted tree with correct support values

Tree Shapes

Evolutionary time



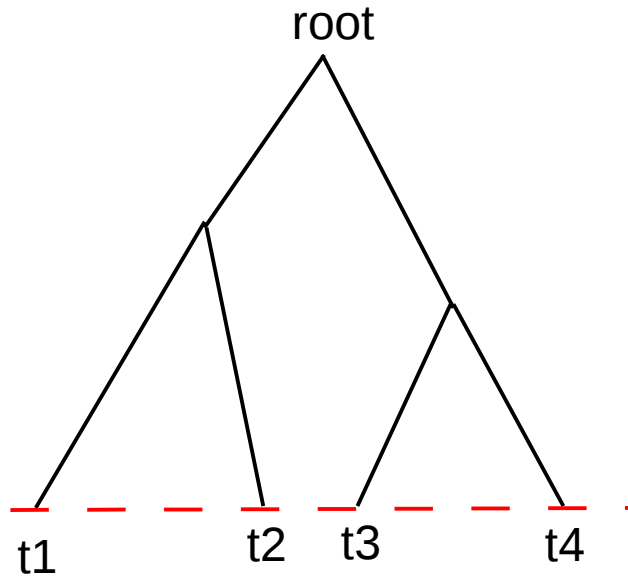
Ultrametric tree



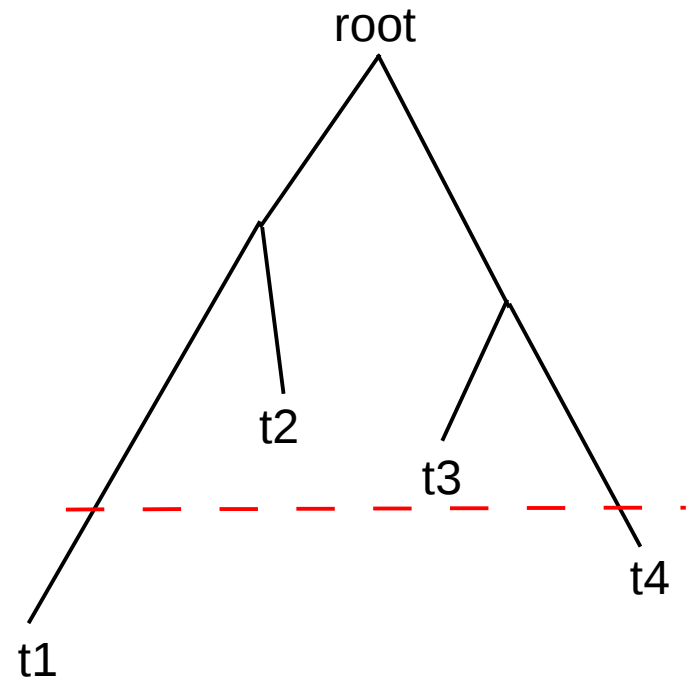
Non-ultrametric tree

Tree Shapes

Evolutionary time



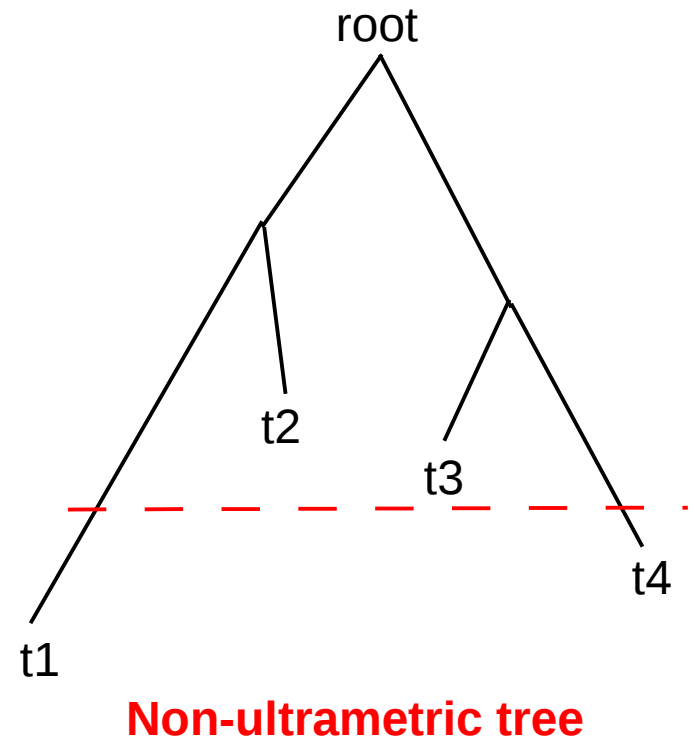
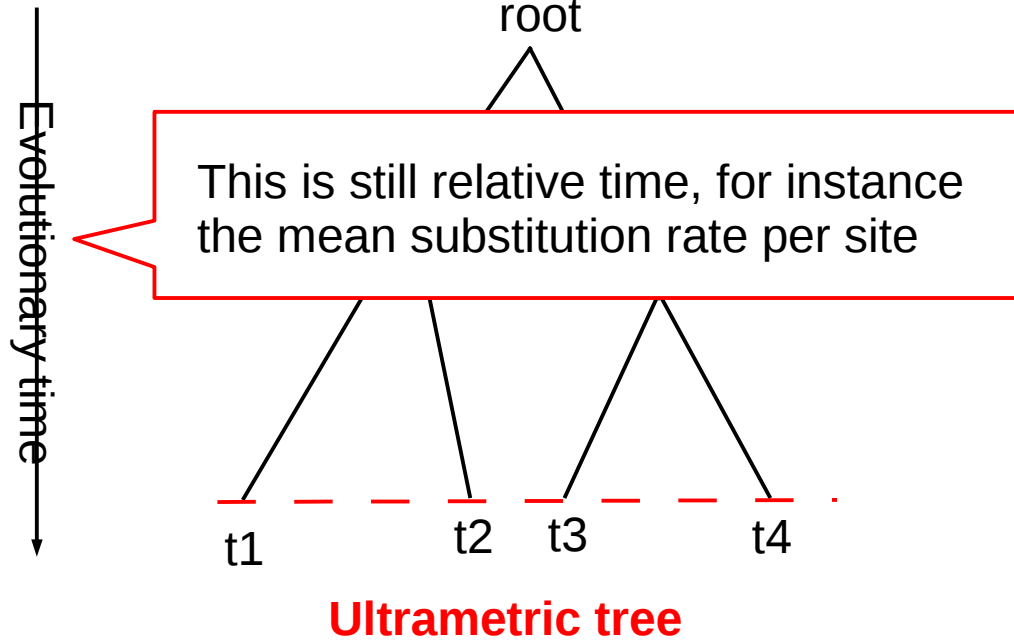
Ultrametric tree



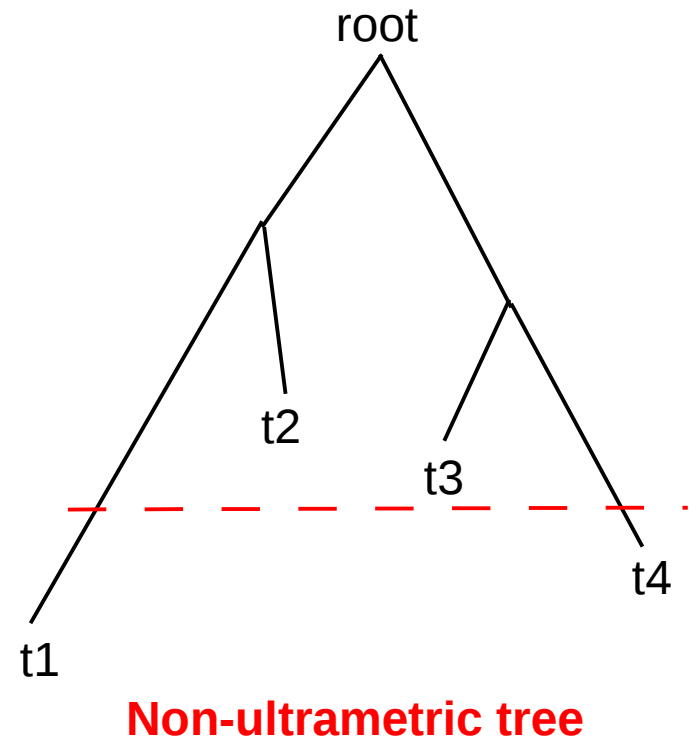
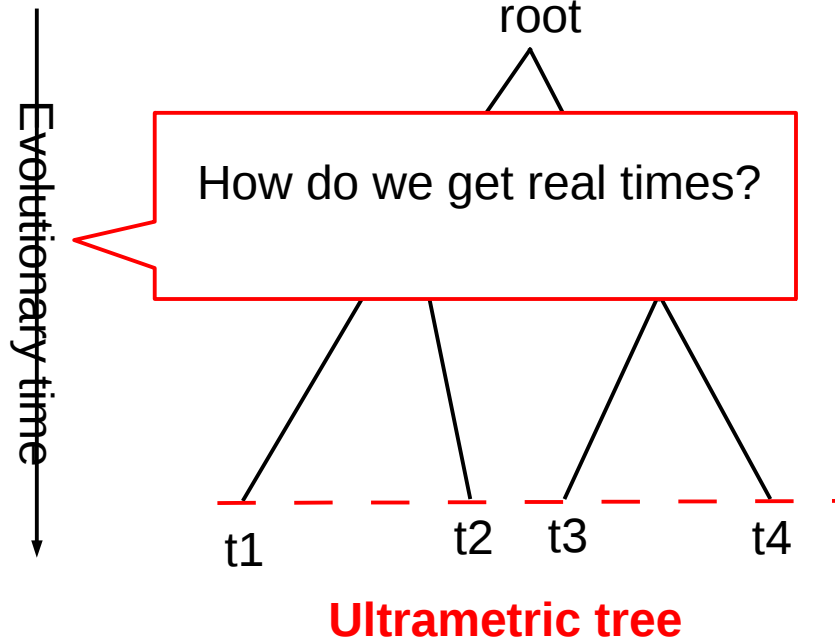
Non-ultrametric tree

Most tree inference models/algorithms/programs produce non-ultrametric trees

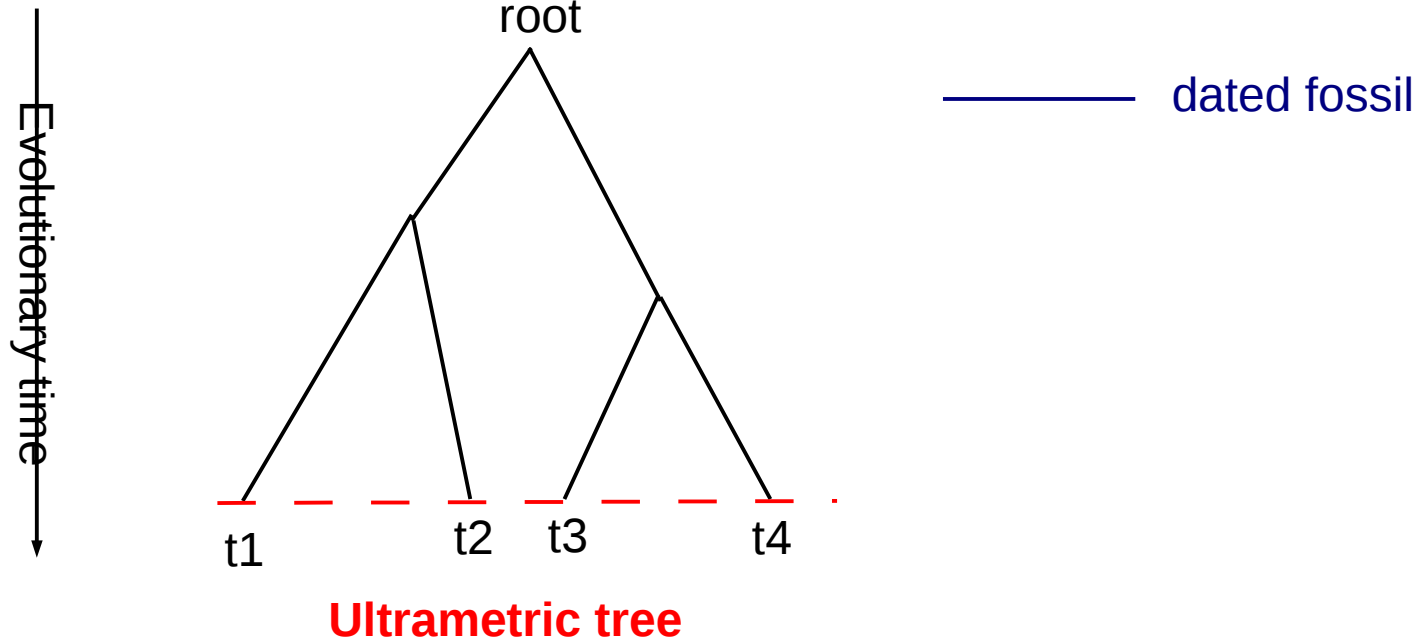
Tree Shapes



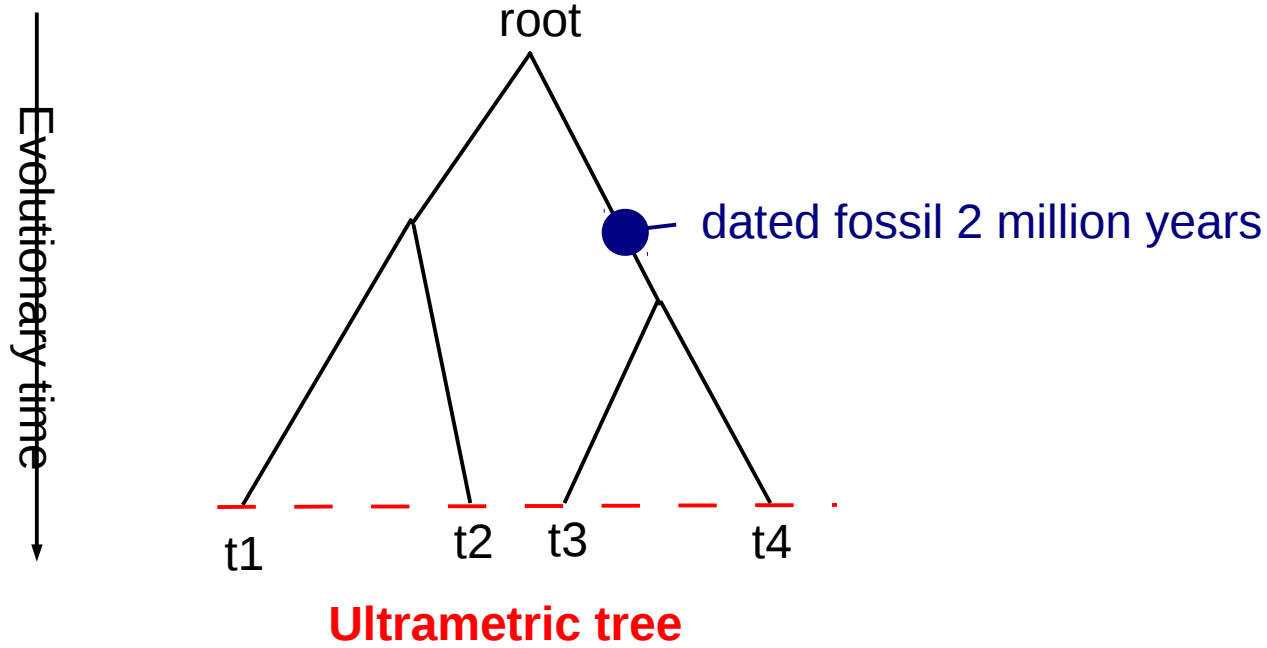
Tree Shapes



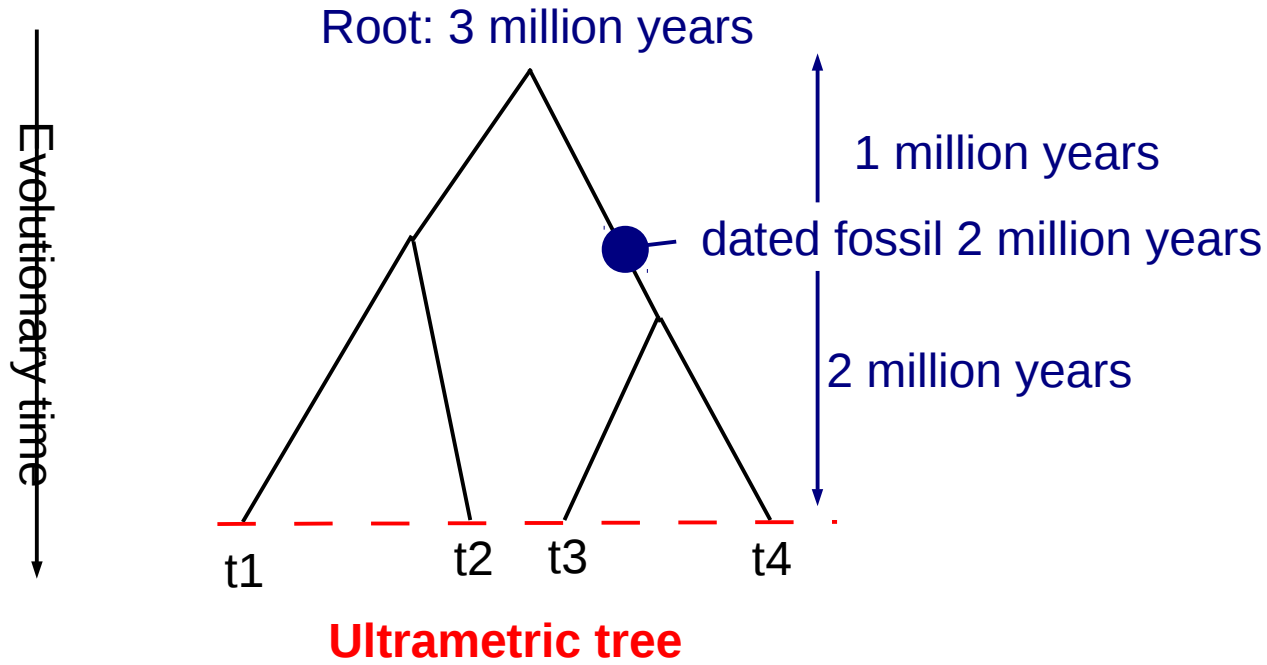
Dating Trees



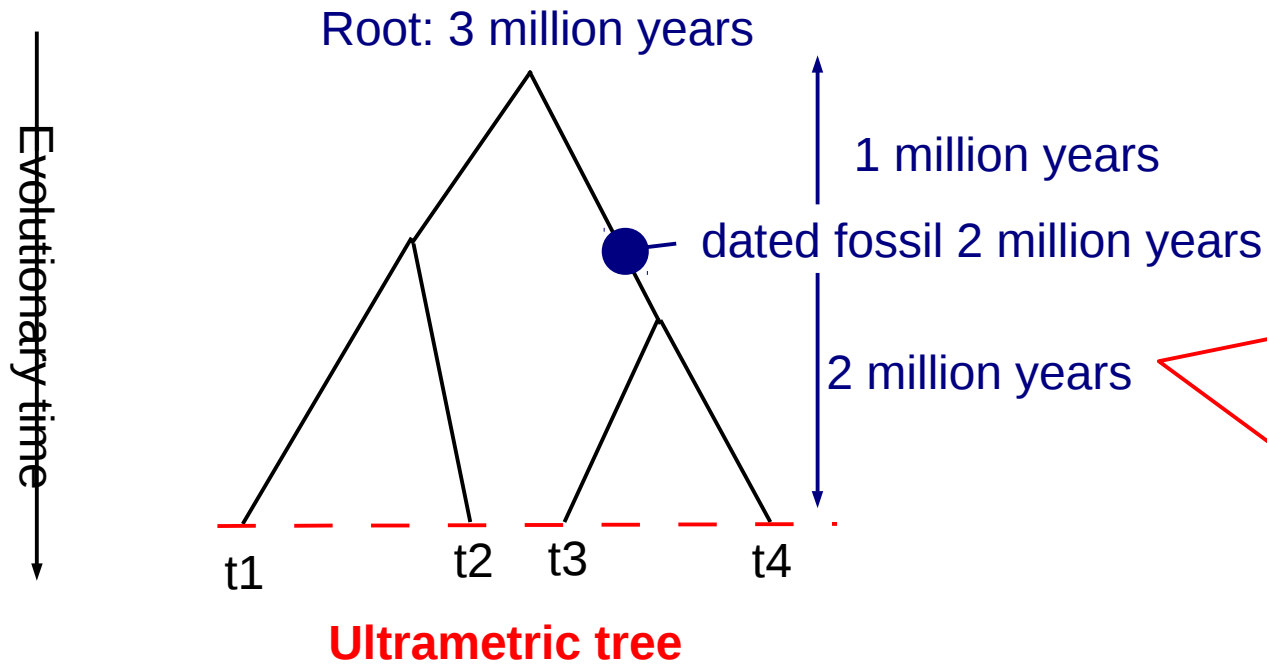
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Dating Trees



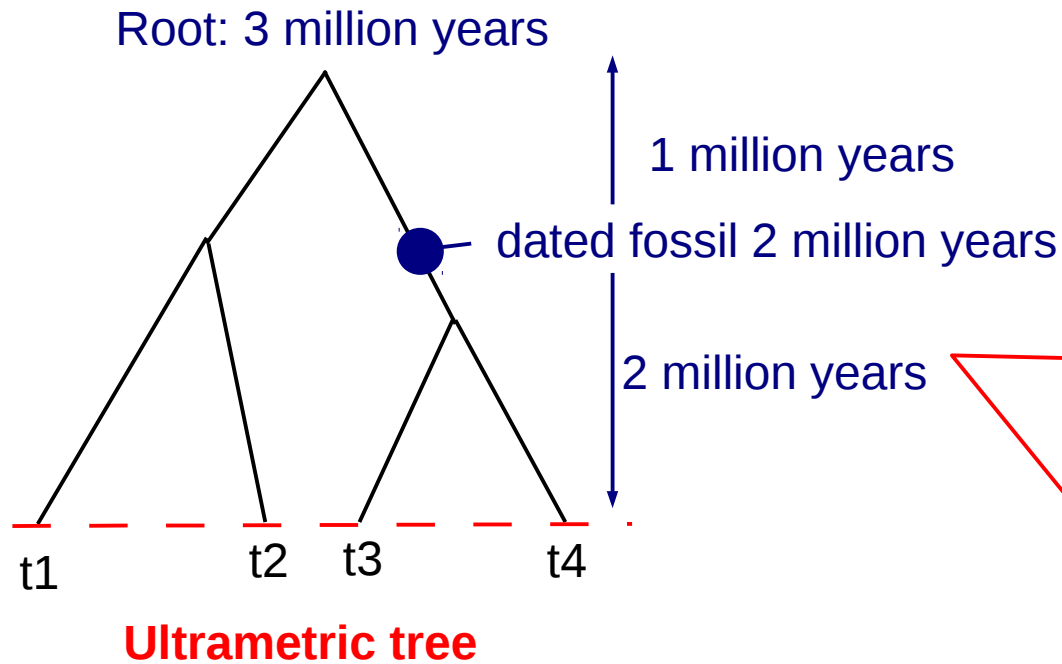
Dating Trees



We need a rooted & ultrametric tree!

- rooting with outgroups
- ultrametricity with programs for *divergence time estimation*
- active research area
- most codes rely on the phylogenetic likelihood function and Bayesian Statistics (MCMC methods)

Dating Trees



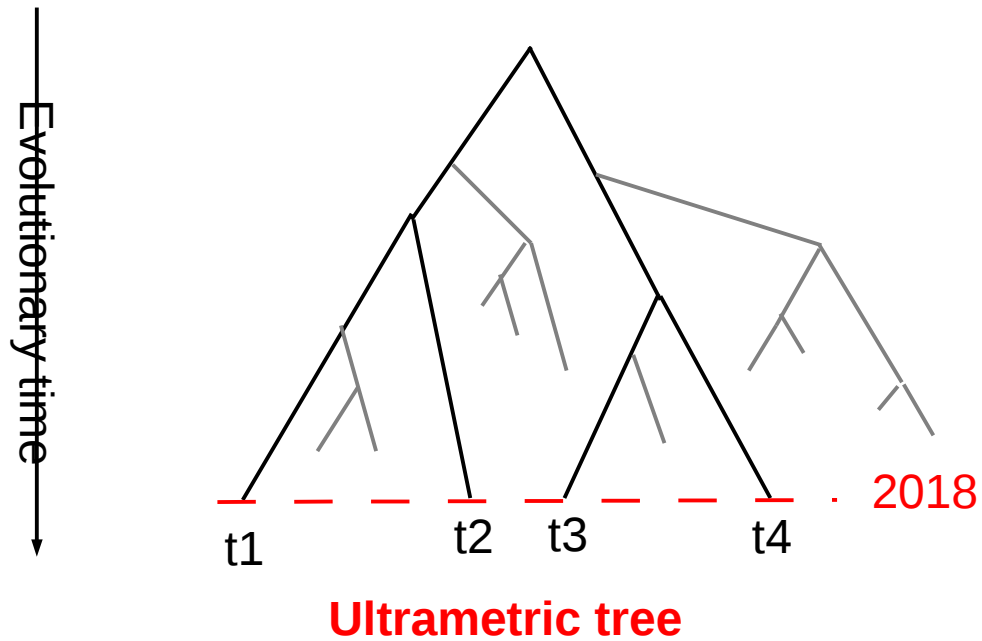
But how do we place the fossil?
→ typically no DNA data available

Fossil placement:
→ ad hoc using empirical knowledge
→ computationally using morphological data

The input for a phylogenetic analysis need not be molecular data!

We can also use sequences of morphological traits ("Merkmale")!

Remember that we deal with extant species!



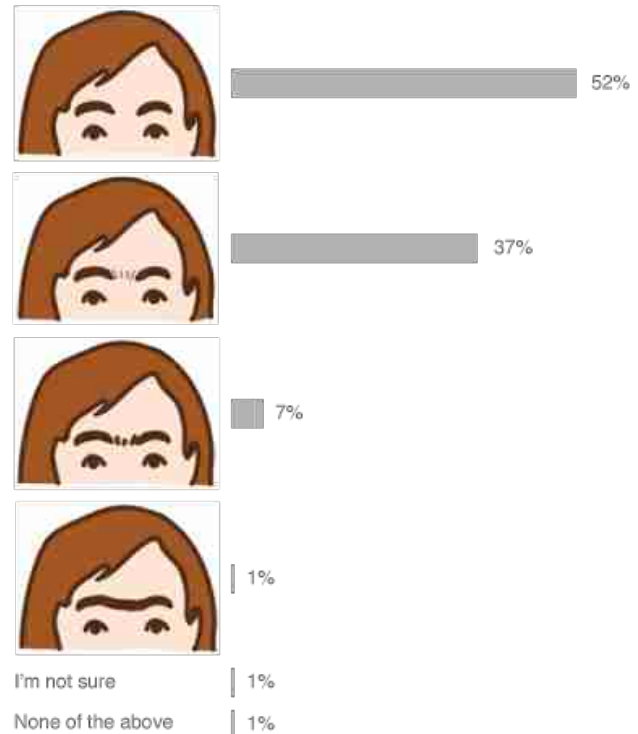
Morphological Traits

t1: 1000
t2: 0100
t3: 0010
T4: 0001

or:

t1: 0
t2: 1
t3: 2
t4: 3

What image best matches the extent of your natural brow line
(without hair removal)?



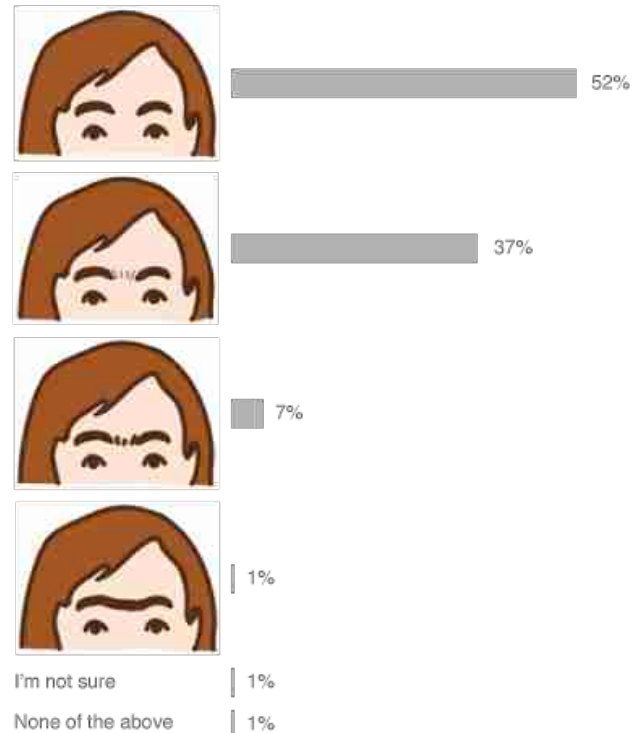
Morphological Traits

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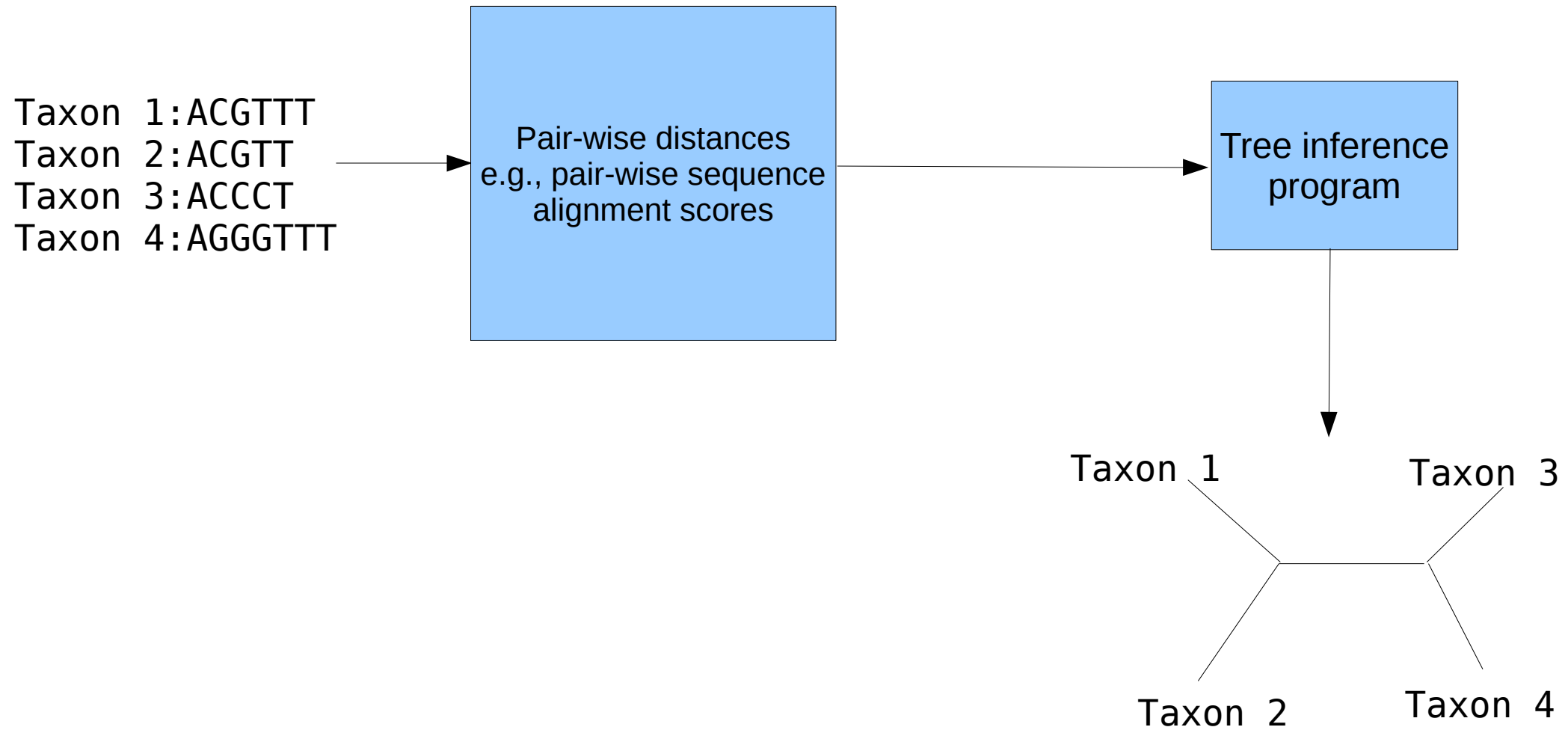
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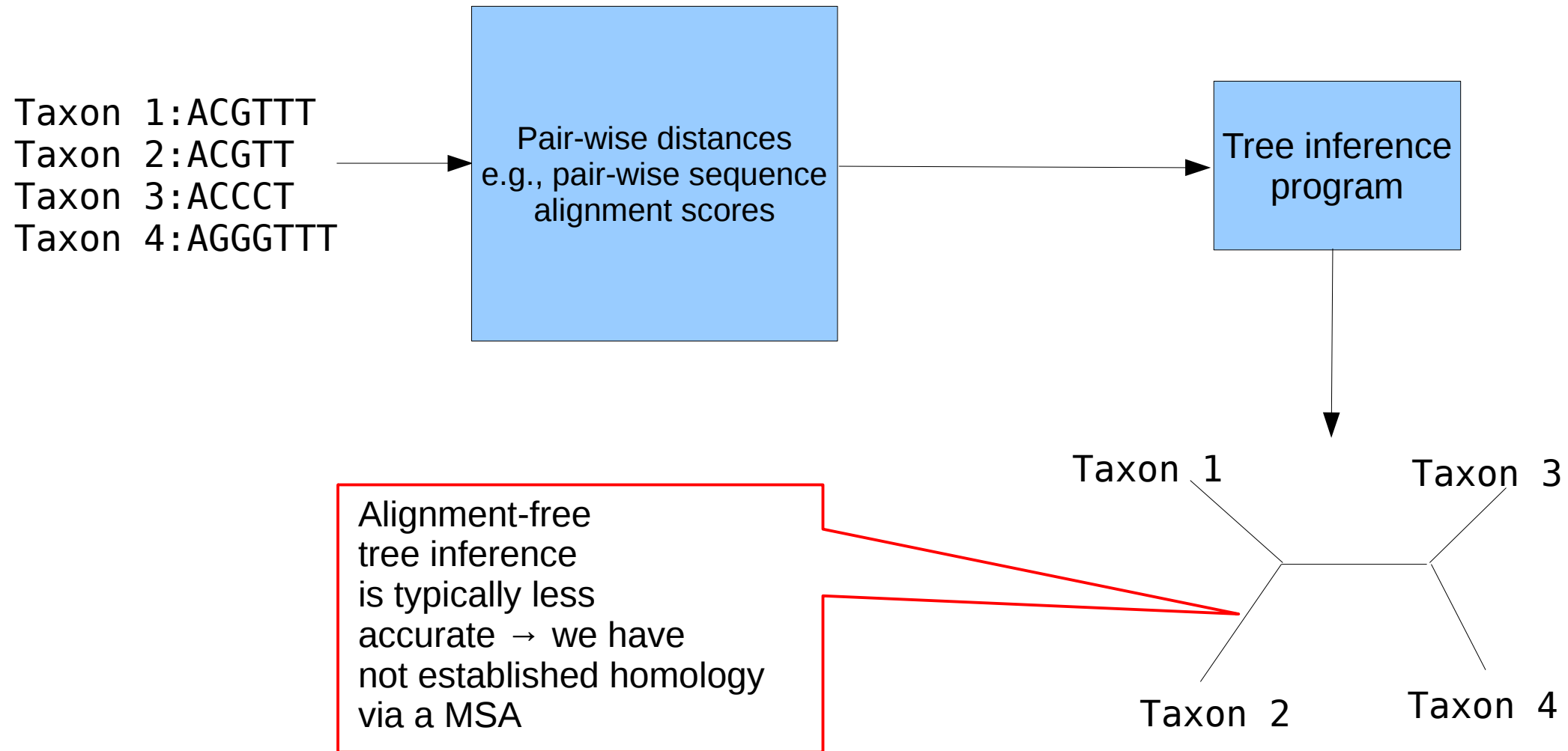


**Traits need not be discrete,
they can also be continuous, e.g., bone ratios**

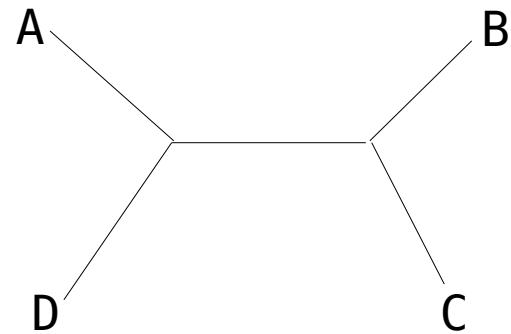
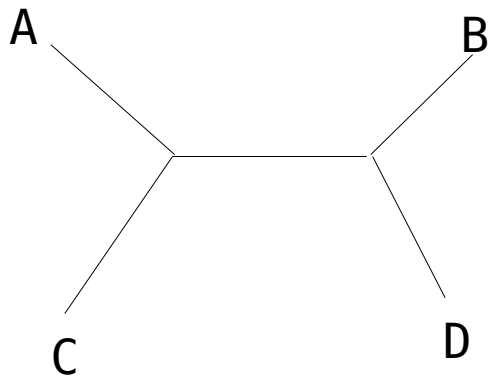
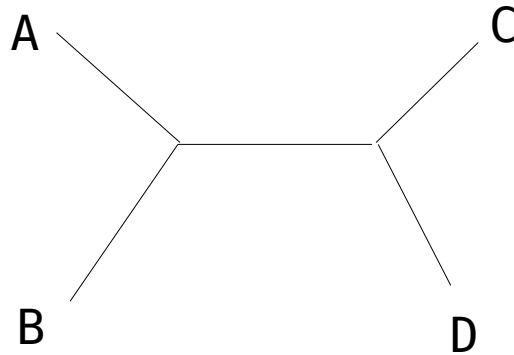
Alignment-Free Tree Inference



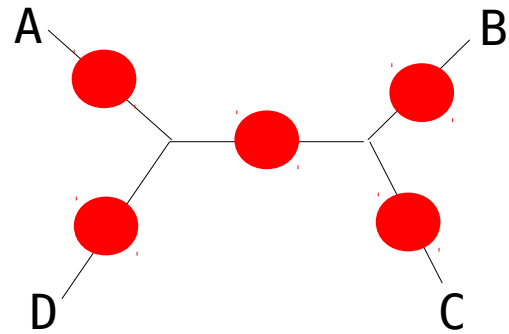
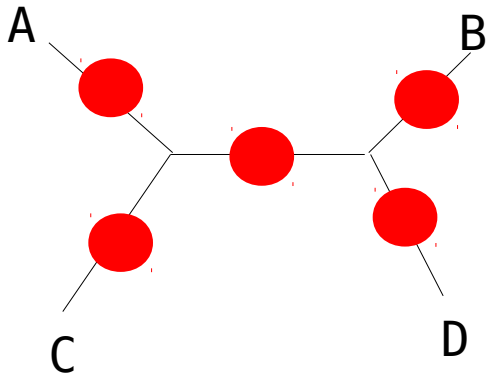
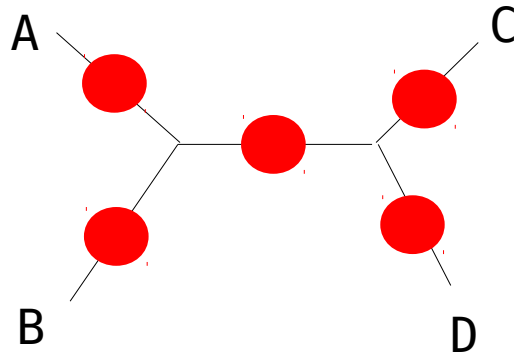
Alignment-Free Tree Inference



How many unrooted 4-taxon trees exist?



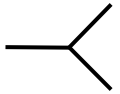
How many rooted 4-taxon trees exist?



Tree Counts

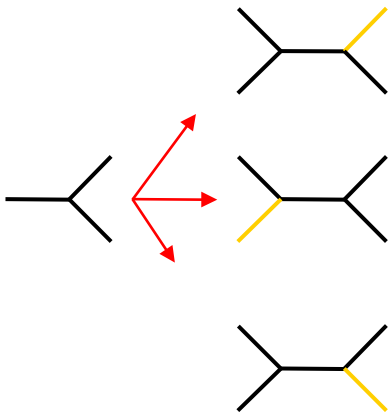
- Unrooted binary trees
 - 4 taxa \rightarrow 3 distinct trees
 - A tree with n taxa has $n-2$ inner nodes
 - And $2n-3$ branches
- Rooted binary trees
 - 4 taxa \rightarrow 3 unrooted trees * 5 branches each (rooting points) = 15 trees
 - $n-1$ inner nodes
 - $2n-2$ branches

The number of trees



3 taxa = 1 tree

The number of trees



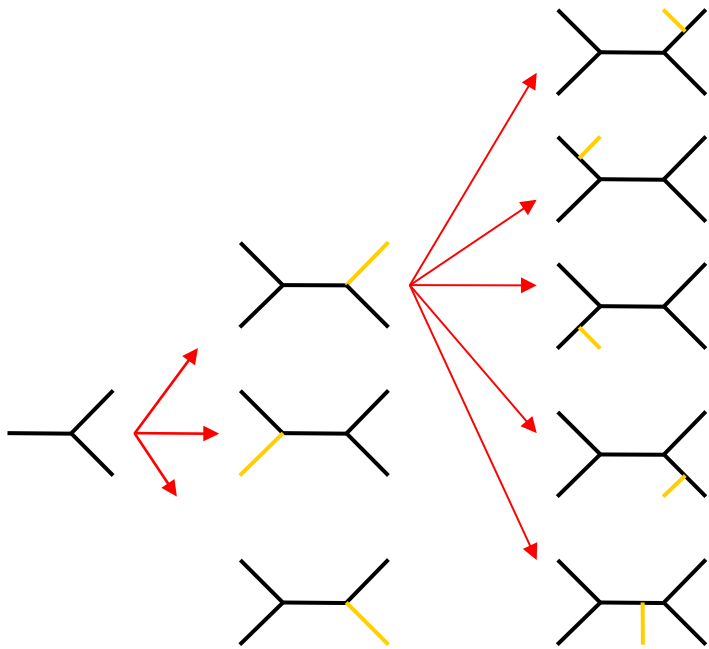
4 taxa: 3 trees

u : # trees of size $4-1 := 1$

v : # branches in a tree of size $4-1 := 3$

Number of unrooted binary trees with 4 taxa: $u * v = 3$

The number of trees



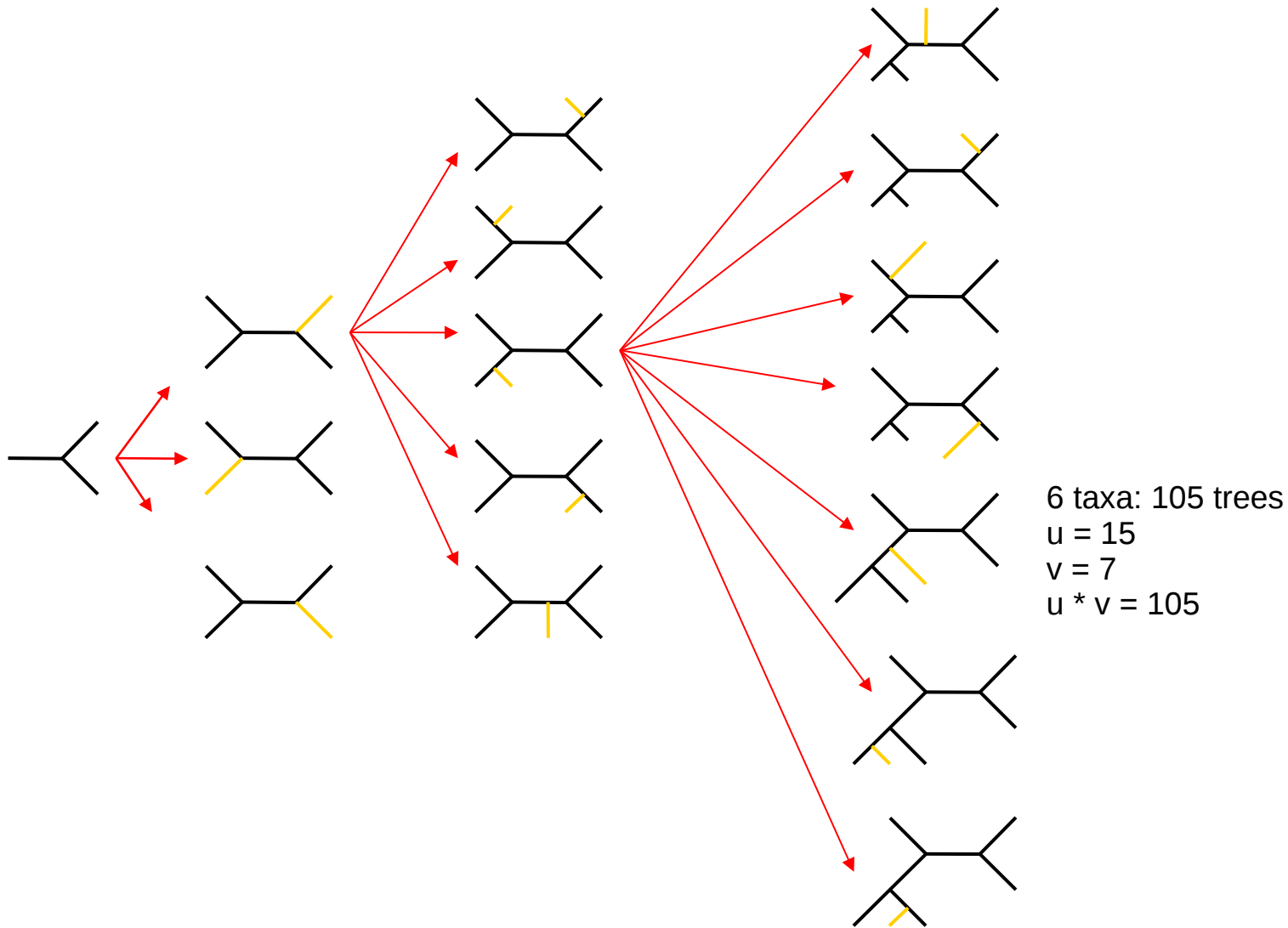
5 taxa: 15 trees

$u = 3$

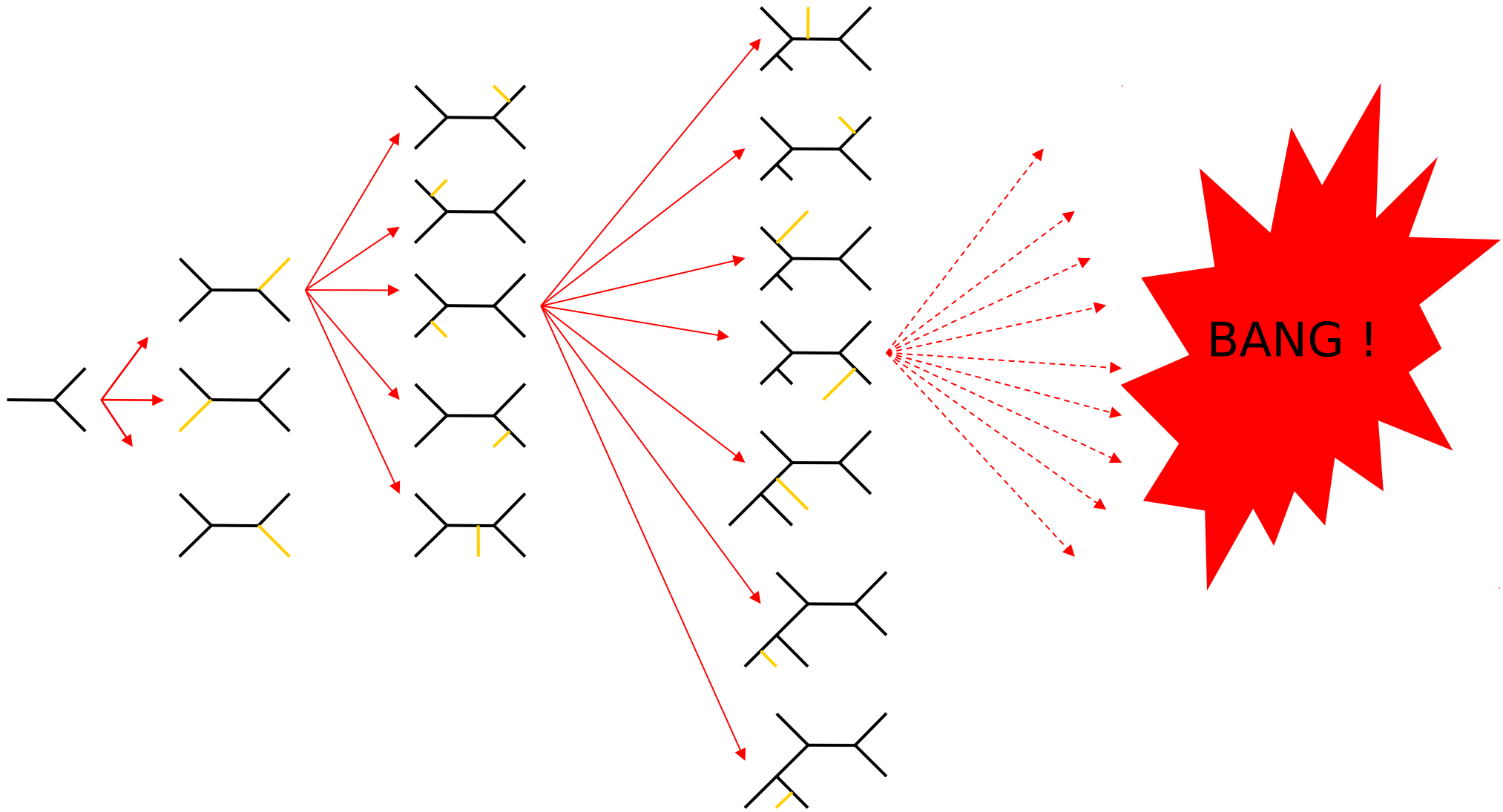
$v = 5$

Number of unrooted trees with 5 taxa: $3 * 5 = 15$

The number of trees



The number of trees explodes!



Some Numbers

Number of Organisms	Number of alternative Trees
3	1
4	3
5	15
6	105
7	945
10	2,027,025
15	7,905,853,580,625
20	$2.21 * 10^{20}$
50	$2.84 * 10^{70}$

Table 2.1: Number of possible trees for phylogenies with 3–50 organisms

Equation for the number of unrooted trees

- Simple proof via induction

$$\prod_{i=1}^n (2i - 5)$$

- The number of rooted trees for n taxa simply is the number of unrooted trees for $n+1$ taxa
- The additional ($n+1^{th}$) taxon represents all possible rootings for all unrooted trees with n taxa

trees with 2000 tips

```
stamatak@exelixis:~/Desktop/GIT/TreeCounter$ ./treeCounter -n 2000
```

GNU GPL tree number calculator released June 2011 by Alexandros Stamatakis

```
Number of unrooted binary trees for 2000 taxa: 30049638174211656151632910065681814981377232074237013089504954043012636525258308210827685996688247000464352735214265634288295
8915023446000631493969130632970436056184861877465482277991223536809233455563199910834597693126756525012899867433187752811401960991631522367030609121735709762379847705467667
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350341796875
```

Approximately 3.00 times 10⁶328

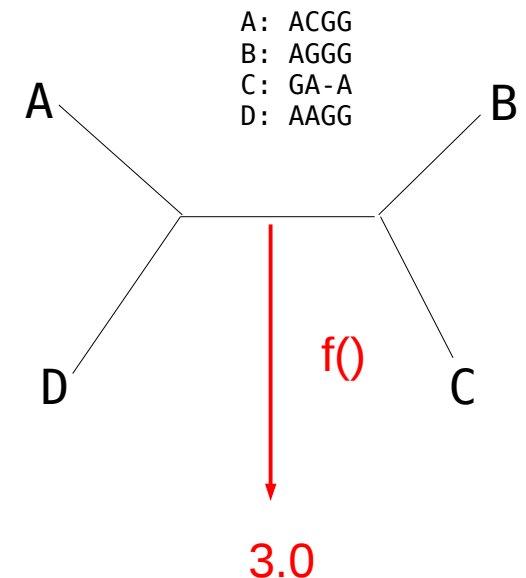
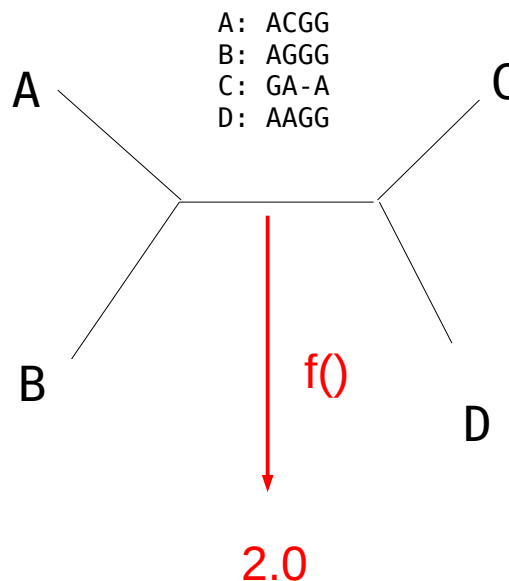
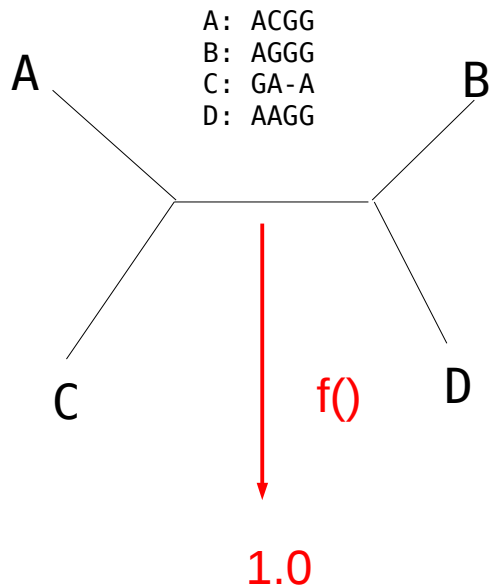
A side-note

The treeCounter tool

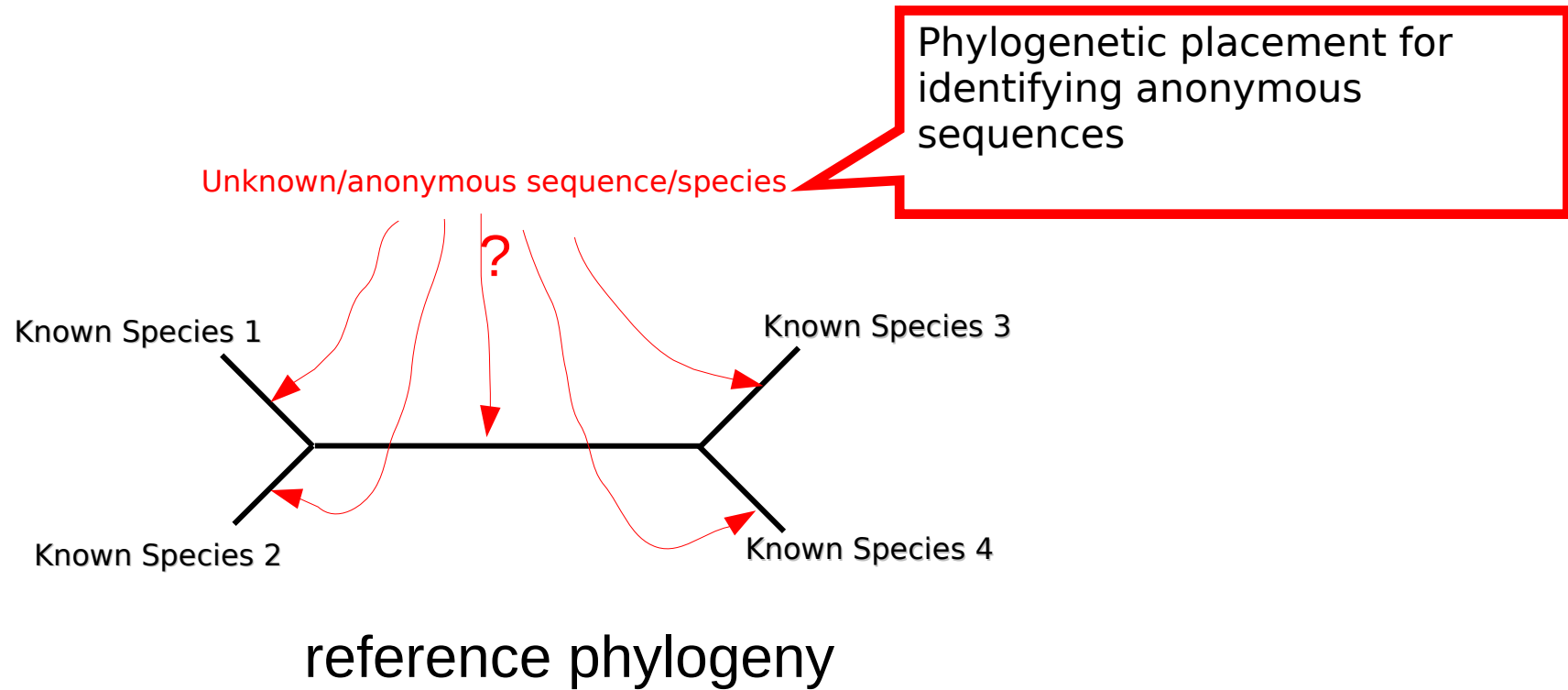
- Evidently, the tree count can not be computed using normal integers
 - we need an arbitrary precision library
 - I used the GNU GMP (Multiple Precision Arithmetic) library
 - treeCounter available as open-source code at <https://github.com/stamatak>
 - Has anybody already used GNU GMP?

Scoring Trees

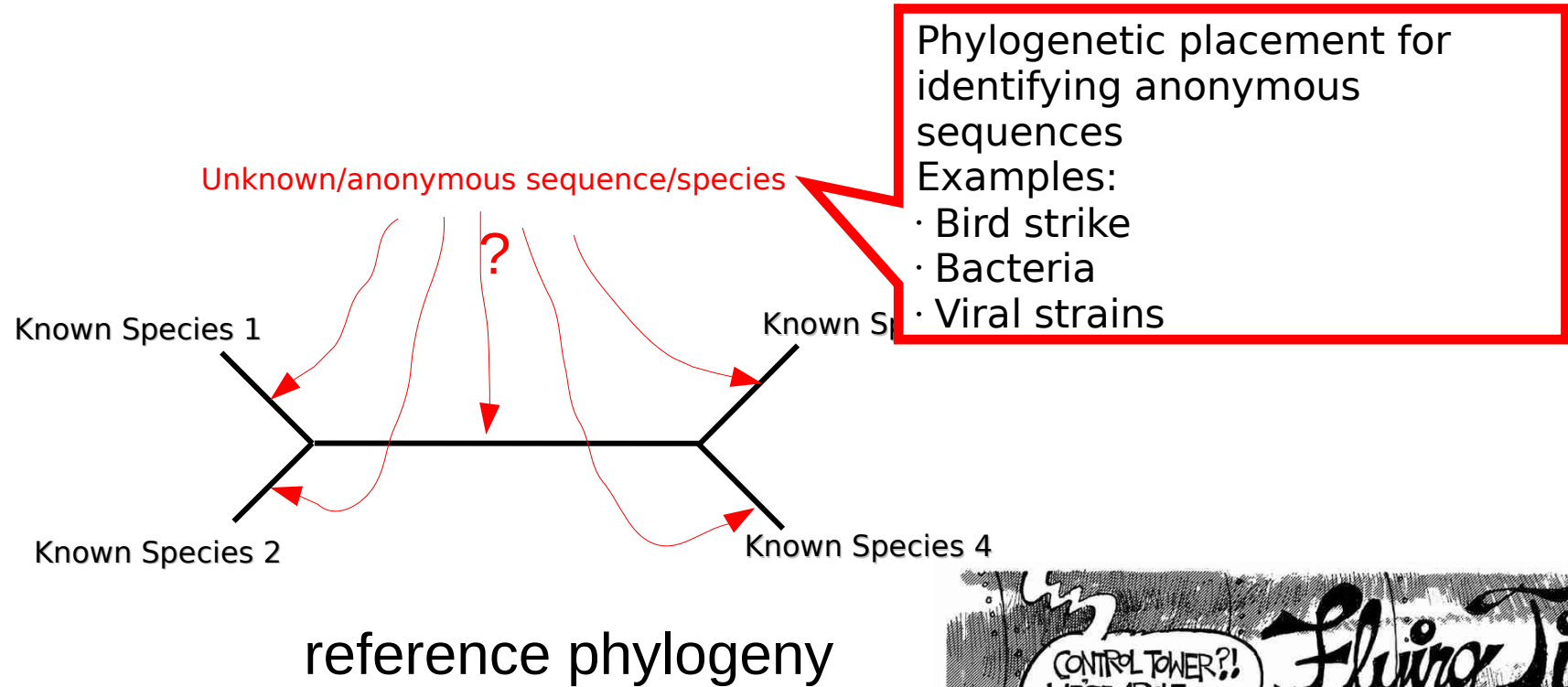
- Now we know how many **unrooted** candidate trees there exist for n taxa
- How do we choose among them?
 - we need some scoring criterion $f()$ to evaluate them
 - finding the optimal tree under most criteria is NP-Hard



What can we do with Phylogenies?

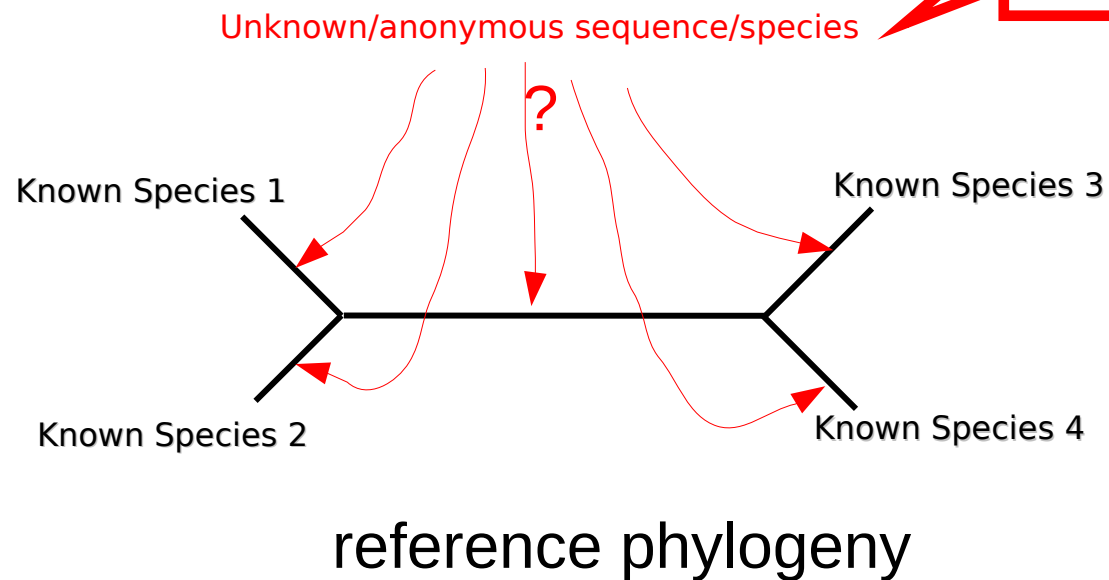


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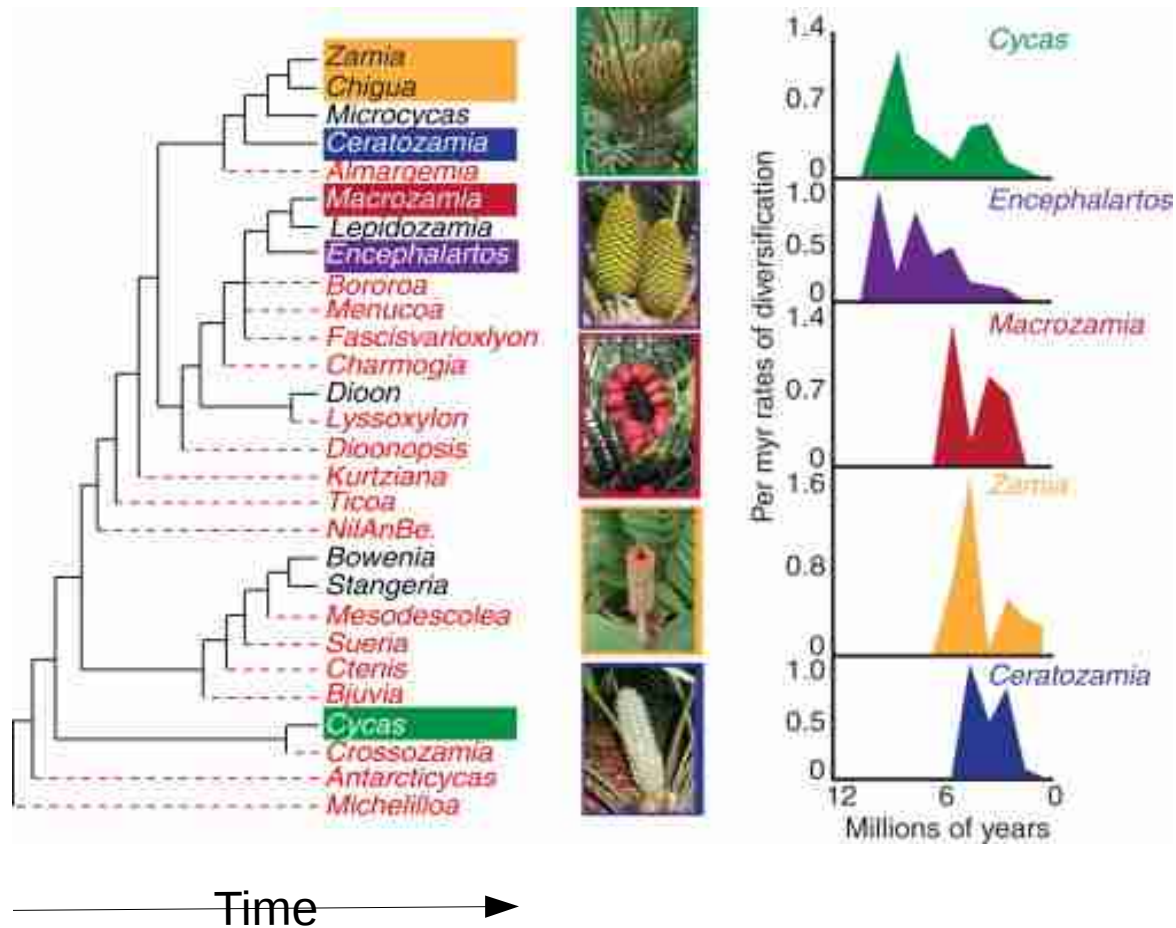


What can we do with Phylogenies?

Note that, this is similar to placing an outgroup into the tree!



Diversification Rates



From: Charles C. Davis, Hanno Schaefer: "Plant Evolution: Pulses of Extinction and Speciation in Gymnosperm Diversity", *Current Biology*, 2011.

Influenza Outbreaks

Host Taxa

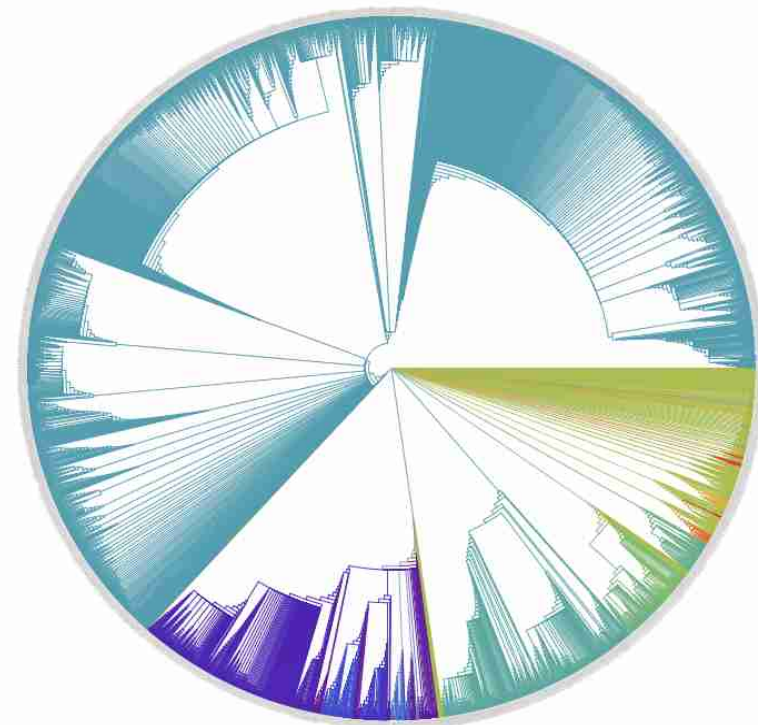
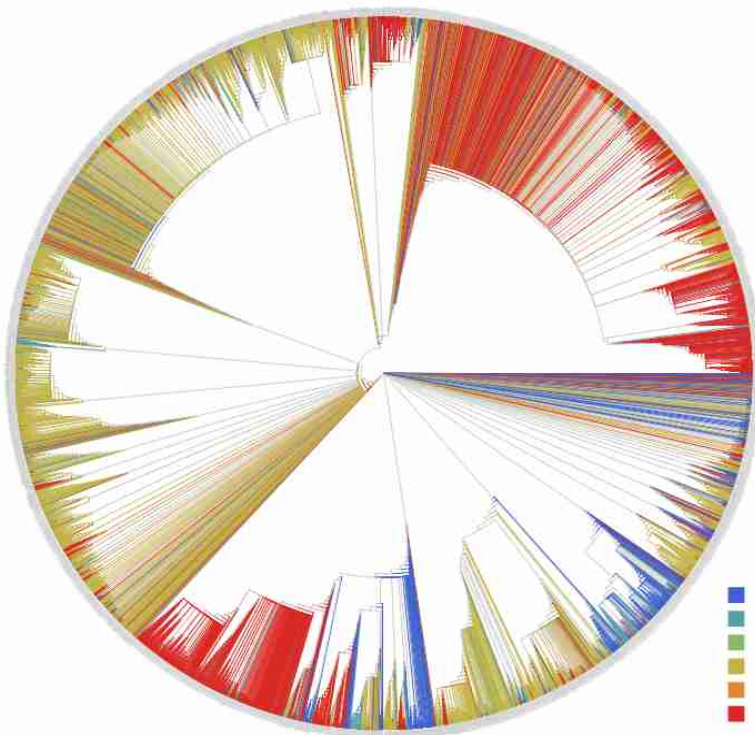
- Galliformes
- Anseriformes
- Passeriformes
- Charadriiformes
- Human
- Columbidae
- Artiodactyla
- Accipitriformes
- Ardeidae
- Carnivora
- Corvidae
- Arthropoda
- Ambiguous



And of course SARS-CoV-2

Phylogenetic analysis of SARS-CoV-2 data is difficult

Benoit Morel^{1,3}, Pierre Barbera^{4,5}, Lucas Czech³, Ben Bettisworth¹, Lukas Hübner^{1,2}, Sarah Lutteropp¹, Dora Serdari¹, Evangelia-Georgia Kostaki⁵, Ioannis Mamais⁶, Alexey M Kozlov¹, Pavlos Pavlidis², Dimitrios Paraskevis³, and Alexandros Stamatakis^{1,2}



- A (103)
- A.1 (393)
- A.2 (60)
- A.3 (68)
- A.4 (9)
- A.5 (28)
- A.6 (6)
- B (15)
- B.1 (3068)
- B.2 (446)
- B.3 (70)
- B.4 (100)
- B.6 (76)
- B.9 (5)
- B.10 (1)
- B.11 (344)
- B.12 (2)
- B.15 (6)
- B.16 (23)
- B.17 (5)
- B.18 (2)
- B.21 (2)
- B.23 (21)
- B.24 (5)
- B.26 (8)
- B.27 (3)

Snakebites

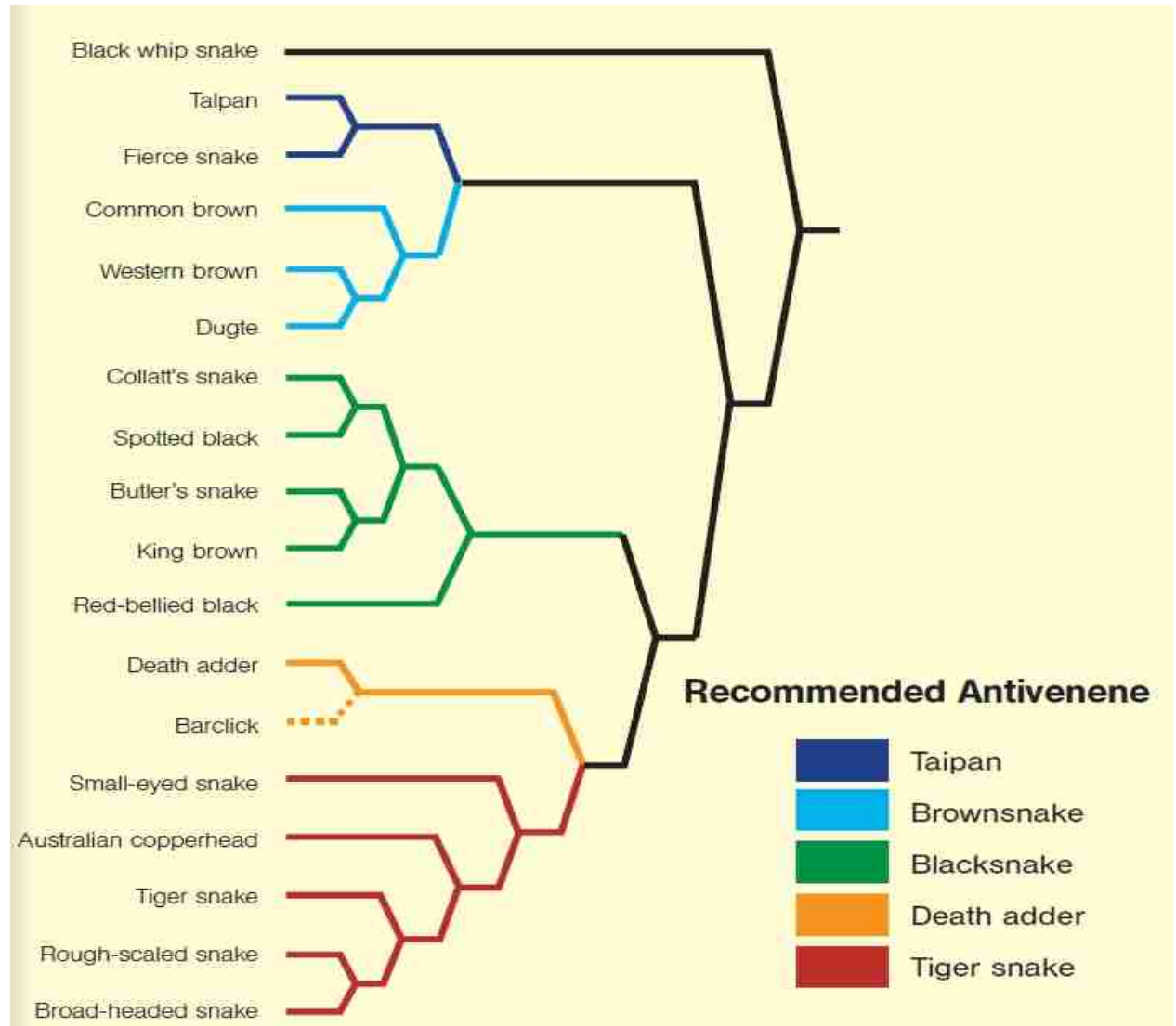
Australia has more poisonous snakes than any other continent, and many people die from **snakebites** each year. Developing **effective antivenins** is thus a **high priority**, but little is known about the venins of most species.

Phylogenetic analysis is helping with this task because **venin properties correlate strongly with evolutionary relationships**.

Although the **red-bellied black snake** looks very different from the **king brown**, it is actually **closely related** and can be treated with the same antivenin.

Conversely, the **western brown** looks very similar to the **king brown**, but it is only distantly related and thus responds best to different antivenin.

The **phylogeny is also predictive**: the recent demonstration that the poorly-known **barclick** is closely related to the **death adder** (orange lineage) predicts that the former is also highly dangerous and might respond to widely-available death adder antivenin.



Snakebites

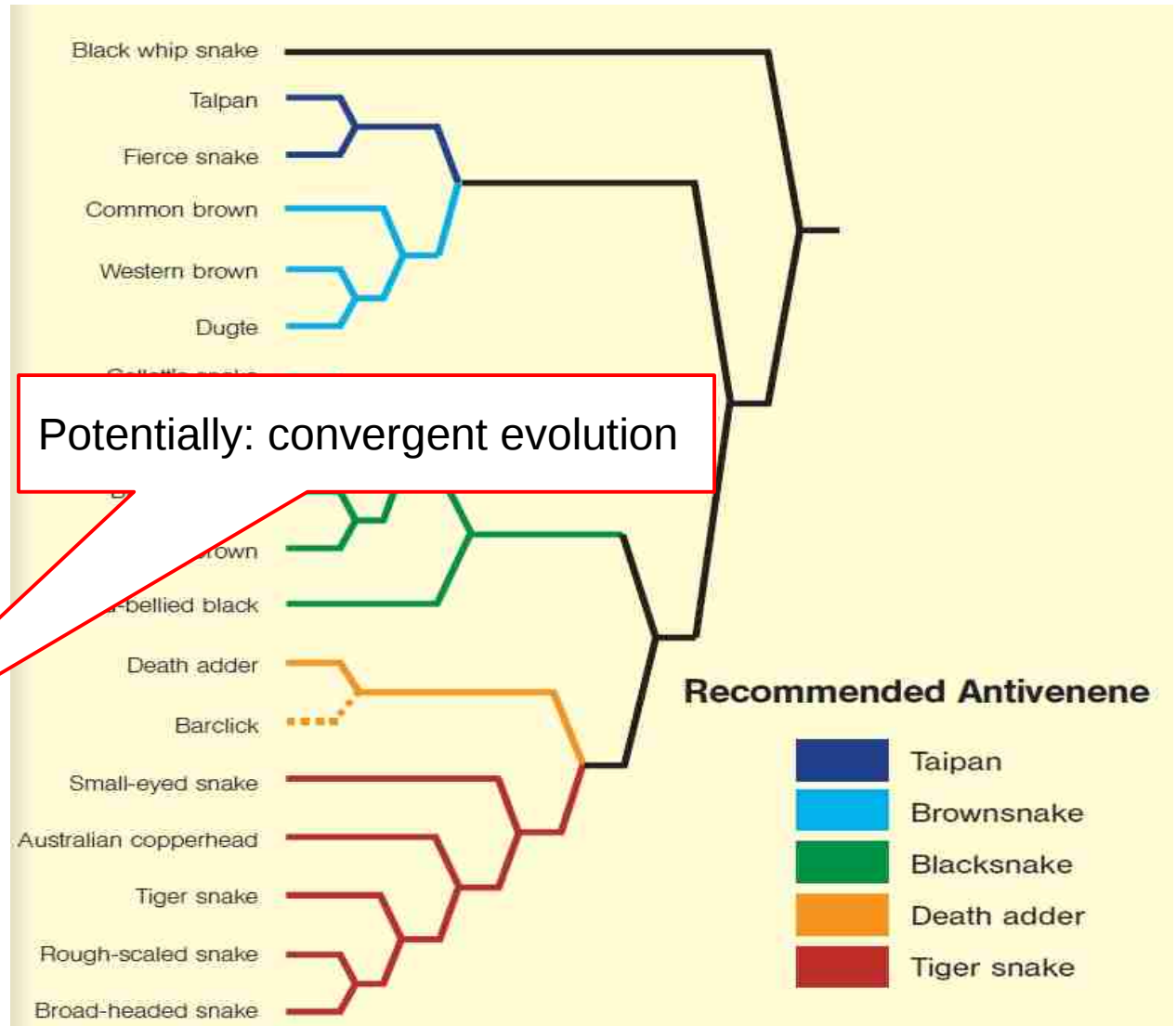
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What can we do with phylogenetic trees?

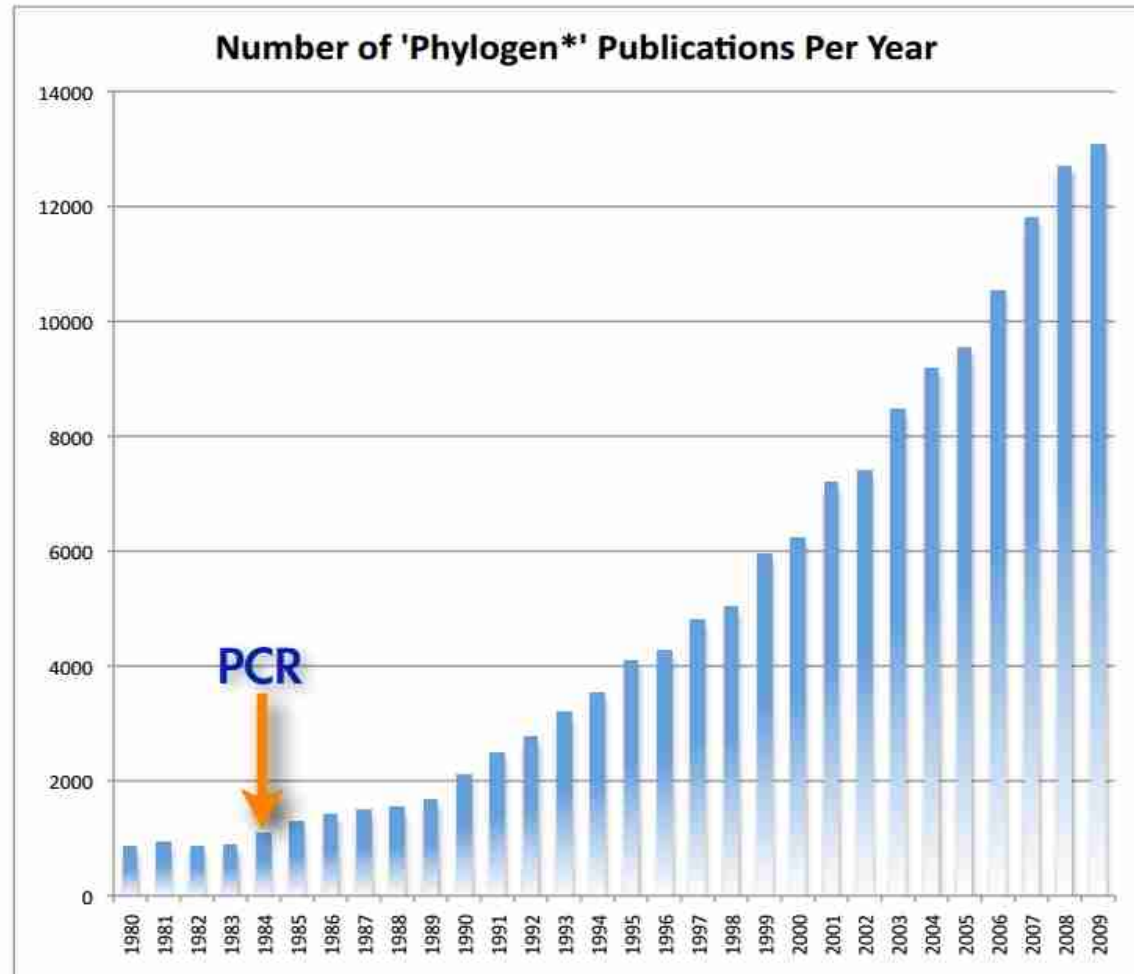
- identifying unknown species
- divergence time estimates
- diversification rates
- viral outbreaks
- forensics → M.L. Metzker, D.P. Mindell, X.M. Liu, R.G. Ptak, R.A. Gibbs, D.M. Hillis: “Molecular evidence of HIV-1 transmission in a criminal case” PNAS: 99(22):14292-7, 2002.

“Nothing in Biology makes sense, except in the light of evolution”

Why this increase in
Phylogenetics papers?

Advances in:

- Sequencing technology
- Hardware
- Methods & Tools



Building Trees

- We distinguish between
 - *Distance-based methods*
 - use MSA to compute a matrix of pair-wise distances
 - build a tree using these distances
 - Heuristics (essentially hierarchical clustering methods)
 - *Neighbor Joining*: NJ
 - *Unweighted Pair Group Method with Arithmetic Mean*: UPGMA
 - least-squares method: explicit optimality criterion
 - *Character-based methods*
 - optimality criteria $f()$ operate directly on the MSA & tree
 - parsimony
 - maximum likelihood
 - Bayesian inference
 - take the current tree topology & MSA to calculate a score
 - the score tells us how well the MSA data fits the tree

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Less accurate,
but faster

Slow, but more
accurate

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Memory-intensive!

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What could be the computational limitation here?

Memory-intensive!

Slow, but more
accurate

Building Trees

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Storing this matrix can become problematic memory-wise

- out-of-core/external memory algorithms
- e.g.: NINJA tool for Neighbor joining

“Large-scale neighbor-joining with ninja”
T Wheeler,
Algorithms in Bioinformatics, 2009

Out-of-core Algorithms

- Definition from Wikipedia:

Out-of-core or External memory algorithms are algorithms that are designed to process data that is too large to fit into a computer's main memory at one time. Such algorithms must be optimized to efficiently fetch and access data stored in slow bulk memory such as hard drive or tape drives.

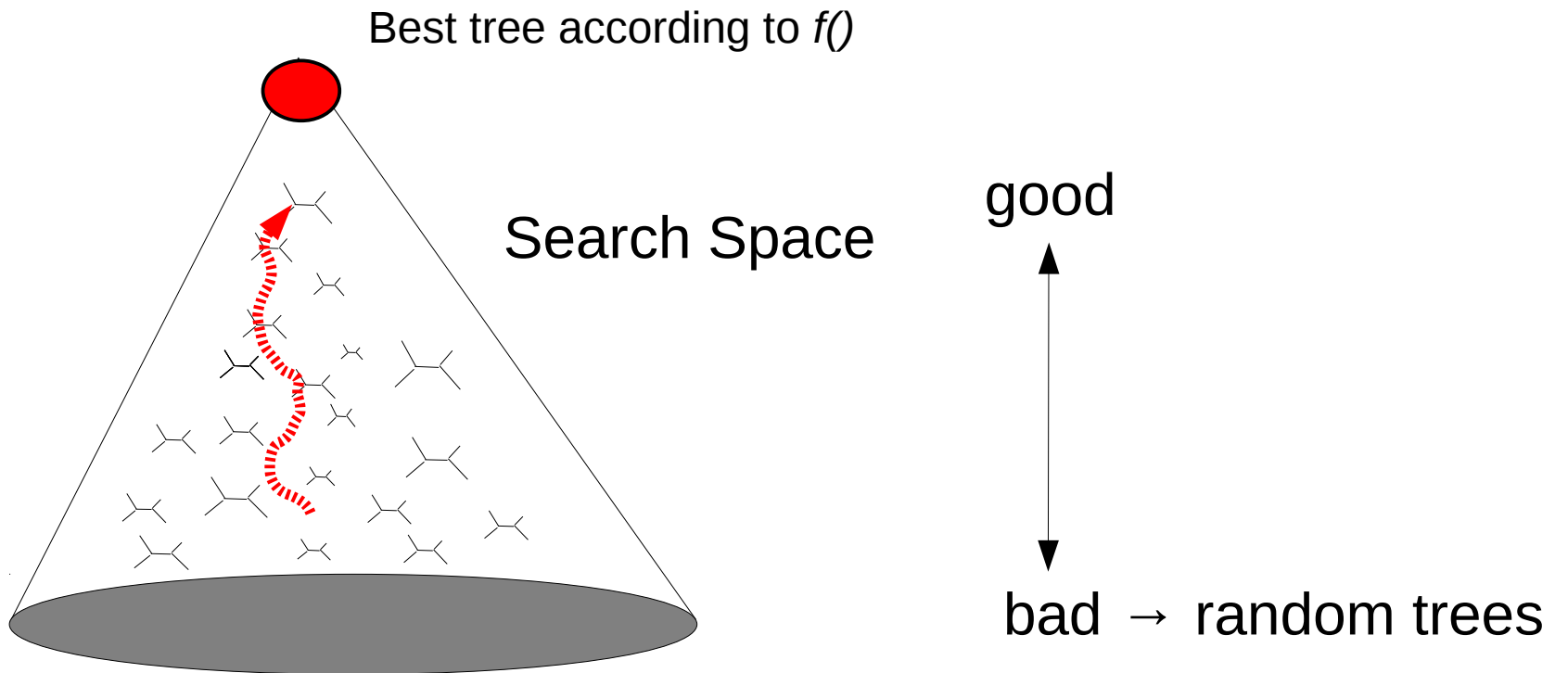
- We do the data transfer RAM ↔ disk explicitly from within the application code by using application-specific knowledge (e.g., about the data access patterns)
- This is to circumvent the paging procedure that would normally be initiated by the OS
- Out-of-core algorithms are typically much faster than the *application-agnostic* paging procedure carried out by the OS
- For an example from phylogenetics see:

Fernando Izquierdo-Carrasco, Alexandros Stamatakis: "Computing the Phylogenetic Likelihood Function Out-of-Core", *IEEE HICOMB 2011 workshop*, Anchorage, USA, May 2011.

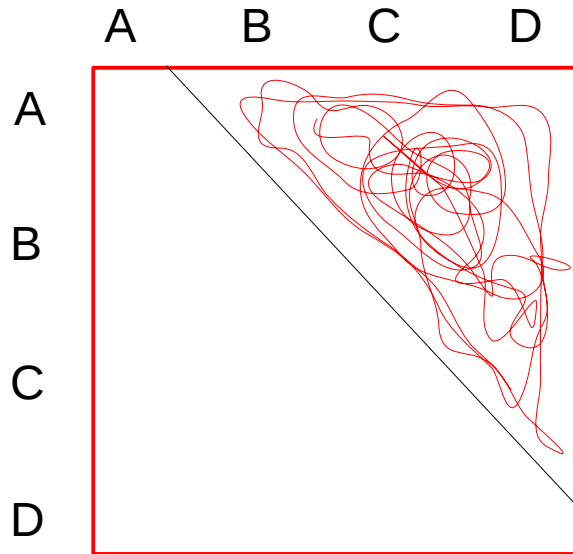
NP-Hardness

- Because of the super-exponential increase in the number of possible trees for n taxa ...
- all interesting criteria on trees are NP-hard:
 - Least squares
 - Parsimony → discrete criterion
 - Likelihood → statistical criterion
 - Bayesian → integrate likelihood over entire tree space

Search Space

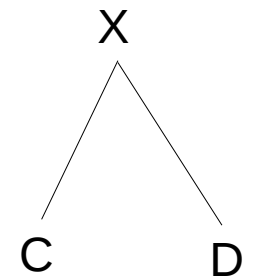
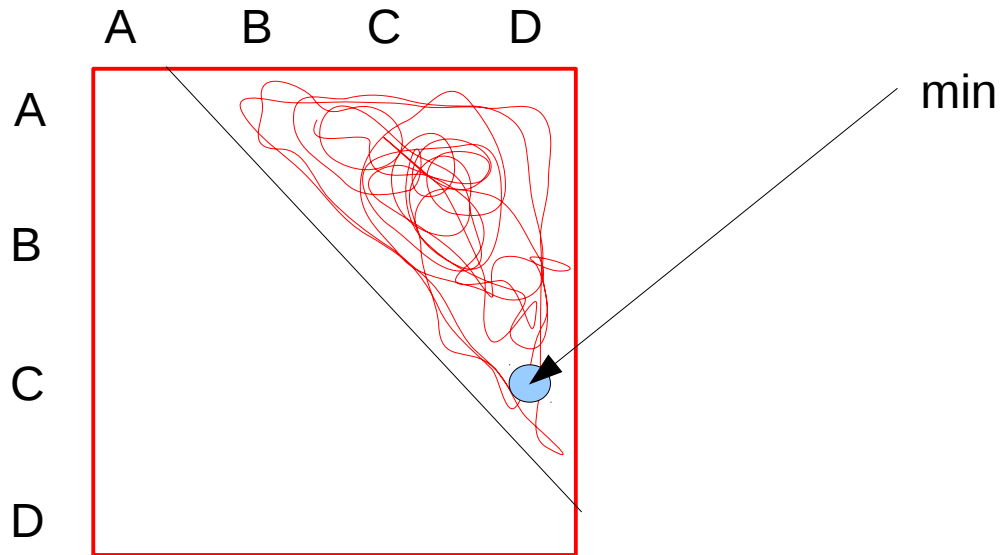


Neighbor Joining → Principle



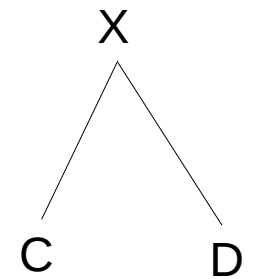
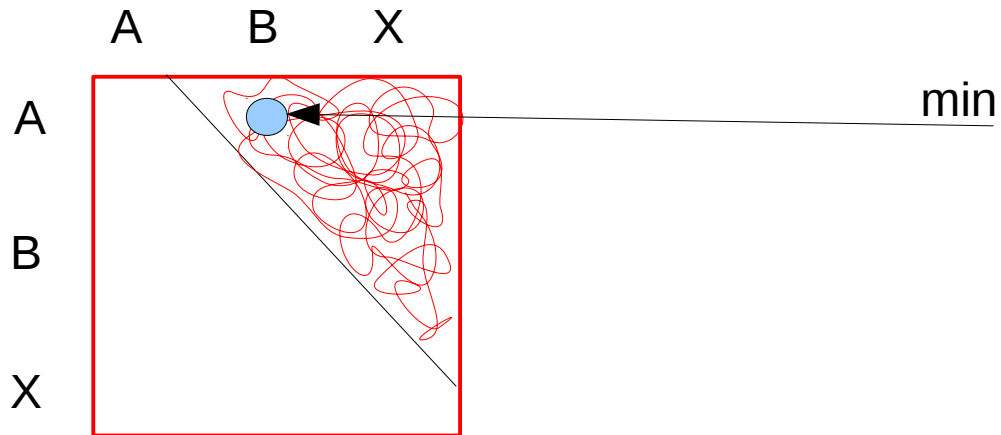
Given a kind of distance matrix $D_{i,j}$ where $i,j=1\dots 4$

Neighbor Joining → Principle



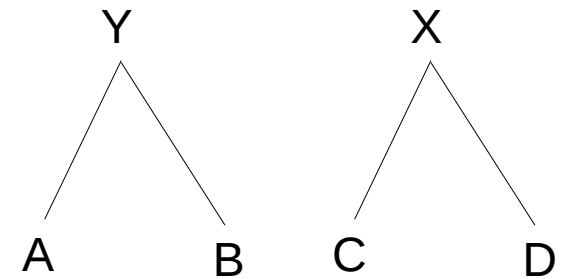
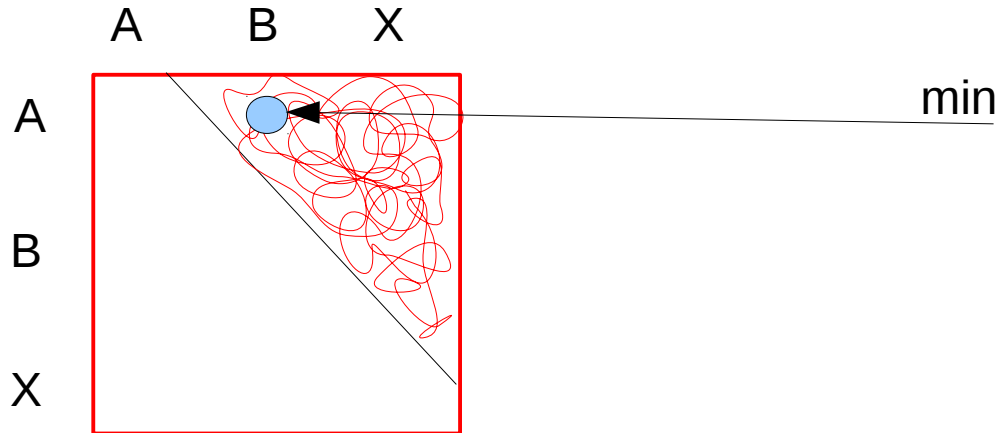
Given a kind of distance matrix $D_{i,j}$ where $i,j=1...4$
Find minimum and merge taxa

Neighbor Joining → Principle



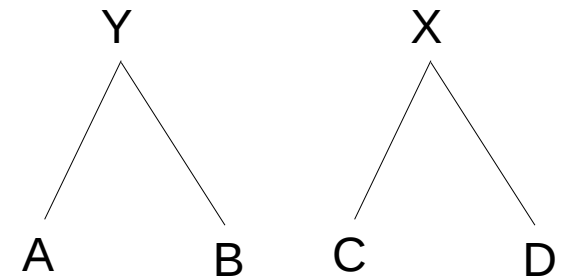
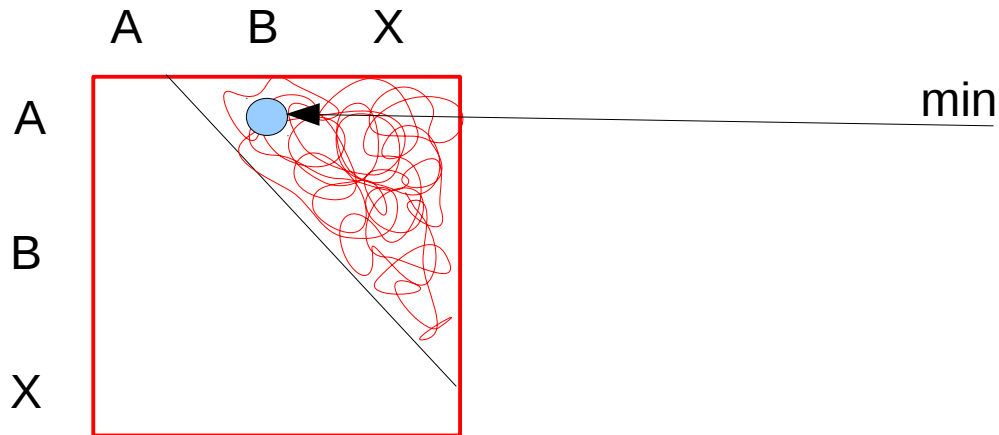
Given a kind of distance matrix $D_{i,j}$ where $i,j=1...4$
Find minimum and merge taxa
Compute a new distance matrix of size $n-1 = 3$
Find minimum

Neighbor Joining → Principle



Given a kind of distance matrix $D_{i,j}$ where $i,j=1\dots4$
Find minimum and merge taxa
Compute a new distance matrix of size $n-1 = 3$
Find minimum and merge taxa

Neighbor Joining → Principle



Given a kind of distance matrix $D_{i,j}$ where $i,j=1\dots4$

Find minimum and merge taxa

Compute a new distance matrix of size $n-1 = 3$

Find minimum and merge taxa

Etc.

Space complexity: $O(n^2)$

Time complexity: $O(n^3)$

Key question: how do we compute distance between X and A or X and B respectively

Neighbor Joining Algorithm

- For each tip compute

$$u_i = \sum_j D_{ij} / (n-2)$$

- this is in principle the average distance to all other tips
- the denominator is $n-2$ instead of n , see below why

- Find the pair of tips, (i, j) for which $D_{ij} - u_i - u_j$ is minimal
- Connect the tips (i, j) to build a new ancestral node X
- The branch lengths from the ancestral node X to i and j are:

$$b_i = 0.5 D_{ij} + 0.5 (u_i - u_j)$$

$$b_j = 0.5 D_{ij} + 0.5 (u_j - u_i)$$

- Update the distance matrix:
 - Compute distance between the new node X and each remaining tip as follows:

$$D_{ij,k} = (D_{ik} + D_{jk} - D_{ij}) / 2$$

- Replace tips i and j by the new node X which is now treated as a tip
- Repeat until only two nodes remain
 - connect the remaining two nodes with each other

Neighbor Joining Algorithm

	A	B	C	D
A	-	17	21	27
B		-	12	18
C			-	14
D				-

Neighbor Joining Algorithm

	A	B	C	D
A	-	17	21	27
B		-	12	18
C			-	14
D				-

Distance matrix, usually denoted as D

i	u_i
A	$(17+21+27)/2=32.5$
B	$(17+12+18)/2=23.5$
C	$(21+12+14)/2=23.5$
D	$(27+18+14)/2=29.5$

Average distance

Neighbor Joining Algorithm

	A	B	C	D
A	-	17	21	27
B		-	12	18
C			-	14
D				-

i	u_i
A	$(17+21+27)/2=32.5$
B	$(17+12+18)/2=23.5$
C	$(21+12+14)/2=23.5$
D	$(27+18+14)/2=29.5$

	A	B	C	D
A	-	-39	-35	-35
B		-	-35	-35
C			-	-39
D				-

$$D_{ij} - u_i - u_j$$

Usually denoted as Q matrix

Neighbor Joining Algorithm

	A	B	C	D
A	-	17	21	27
B		-	12	18
C			-	14
D				-

i	u_i
A	$(17+21+27)/2=32.5$
B	$(17+12+18)/2=23.5$
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A	-	-39	-35	-35
B		-	-35	-35
C			-	-39
D				-

$$D_{ij} - u_i - u_j$$

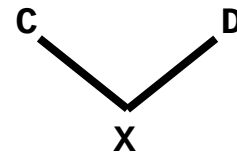
Neighbor Joining Algorithm

	A	B	C	D
A	-	17	21	27
B		-	12	18
C			-	14
D				-

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	A	B	C	D
A	-	-39	-35	-35
B		-	-35	-35
C			-	-39
D				-

$$D_{ij} - u_i - u_j$$



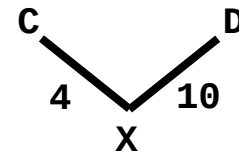
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C			-	14
D				-

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	A	B	C	D
A	-	-39	-35	-35
B		-	-35	-35
C			-	-39
D				-

$$D_{ij} - u_i - u_j$$

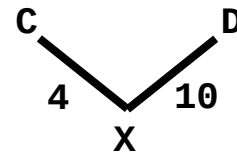


$$b_c = 0.5 \times 14 + 0.5 \times (23.5 - 29.5) = 4$$

$$b_d = 0.5 \times 14 + 0.5 \times (29.5 - 23.5) = 10$$

Neighbor Joining Algorithm

	A	B	C	D	X
A	-	17	21	27	
B		-	12	18	
C			-	14	
D				-	
X					-

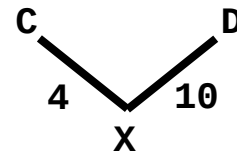


Neighbor Joining Algorithm

	A	B	C	D	X
A	-	17	21	27	
B		-	12	18	
C			-	14	
D				-	
X					-

$$\begin{aligned}D_{XA} &= (D_{CA} + D_{DA} - D_{CD})/2 \\ &= (21 + 27 - 14)/2 \\ &= 17\end{aligned}$$

$$\begin{aligned}D_{XB} &= (D_{CB} + D_{DB} - D_{CD})/2 \\ &= (12 + 18 - 14)/2 \\ &= 8\end{aligned}$$

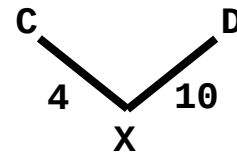


Neighbor Joining Algorithm

	A	B	C	D	X
A	-	17	21	27	17
B		-	12	18	8
C			-	14	
D				-	
X					-

$$\begin{aligned}D_{XA} &= (D_{CA} + D_{DA} - D_{CD})/2 \\ &= (21 + 27 - 14)/2 \\ &= 17\end{aligned}$$

$$\begin{aligned}D_{XB} &= (D_{CB} + D_{DB} - D_{CD})/2 \\ &= (12 + 18 - 14)/2 \\ &= 8\end{aligned}$$

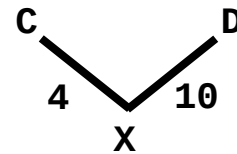


Neighbor Joining Algorithm

	A	B	X
A	-	17	17
B		-	8
X			-

$$\begin{aligned}D_{XA} &= (D_{CA} + D_{DA} - D_{CD})/2 \\ &= (21 + 27 - 14)/2 \\ &= 17\end{aligned}$$

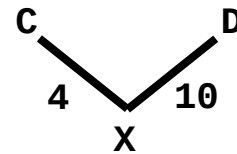
$$\begin{aligned}D_{XB} &= (D_{CB} + D_{DB} - D_{CD})/2 \\ &= (12 + 18 - 14)/2 \\ &= 8\end{aligned}$$



Neighbor Joining Algorithm

	A	B	X
A	-	17	17
B		-	8
X			-

i	u_i
A	$(17+17)/1 = 34$
B	$(17+8)/1 = 25$
X	$(17+8)/1 = 25$

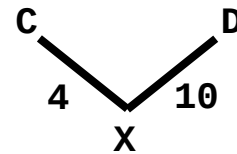


Neighbor Joining Algorithm

	A	B	X
A	-	17	17
B		-	8
X			-

i	u_i
A	$(17+17)/1 = 34$
B	$(17+8)/1 = 25$
X	$(17+8)/1 = 25$

	A	B	X
A	-	-42	-28
B		-	-28
X			-



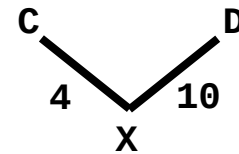
$$D_{ij} - u_i - u_j$$

Neighbor Joining Algorithm

	A	B	X
A	-	17	17
B		-	8
X			-

i	u_i
A	$(17+17)/1 = 34$
B	$(17+8)/1 = 25$
X	$(17+8)/1 = 25$

	A	B	X
A	-	-42	-28
B		-	-28
X			-



$$D_{ij} - u_i - u_j$$

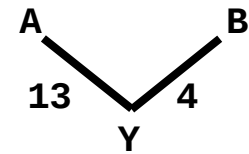
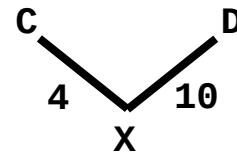
Neighbor Joining Algorithm

	A	B	X
A	-	17	17
B		-	8
X			-

i	u_i
A	$(17+17)/1 = 34$
B	$(17+8)/1 = 25$
X	$(17+8)/1 = 25$

	A	B	X
A	-	-42	-28
B		-	-28
X			-

$$D_{ij} - u_i - u_j$$

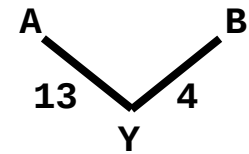
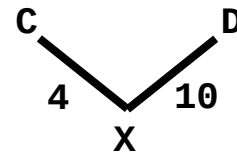


$$b_A = 0.5 \times 17 + 0.5 \times (34 - 25) = 13$$

$$b_D = 0.5 \times 17 + 0.5 \times (25 - 34) = 4$$

Neighbor Joining Algorithm

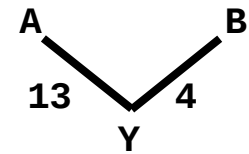
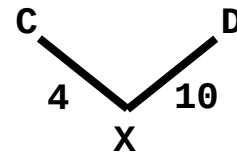
	A	B	X	Y
A	-	17	17	
B		-	8	
X			-	
Y				-



Neighbor Joining Algorithm

	A	B	X	Y
A	-	17	17	
B		-	8	
X			-	4
Y				-

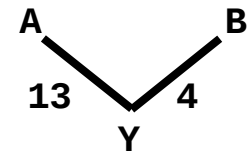
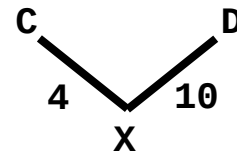
$$\begin{aligned}D_{YX} &= (D_{AX} + D_{BX} - D_{AB})/2 \\ &= (17 + 8 - 17)/2 \\ &= 4\end{aligned}$$



Neighbor Joining Algorithm

	X	Y
X	-	4
Y		-

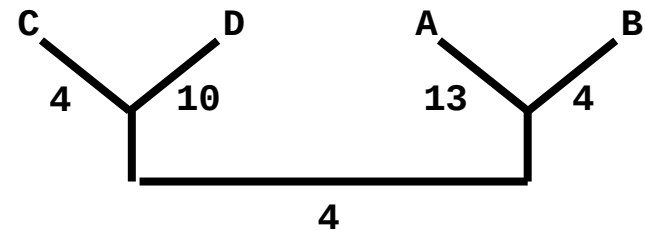
$$\begin{aligned}D_{YX} &= (D_{AX} + D_{BX} - D_{AB})/2 \\ &= (17 + 8 - 17)/2 \\ &= 4\end{aligned}$$



Neighbor Joining Algorithm

	X	Y
X	-	4
Y		-

$$\begin{aligned}D_{YX} &= (D_{AX} + D_{BX} - D_{AB})/2 \\ &= (17 + 8 - 17)/2 \\ &= 4\end{aligned}$$



Neighbor Joining Algorithm

	A	B	C	D
A	-	17	21	27
B		-	12	18
C			-	14
D				-

